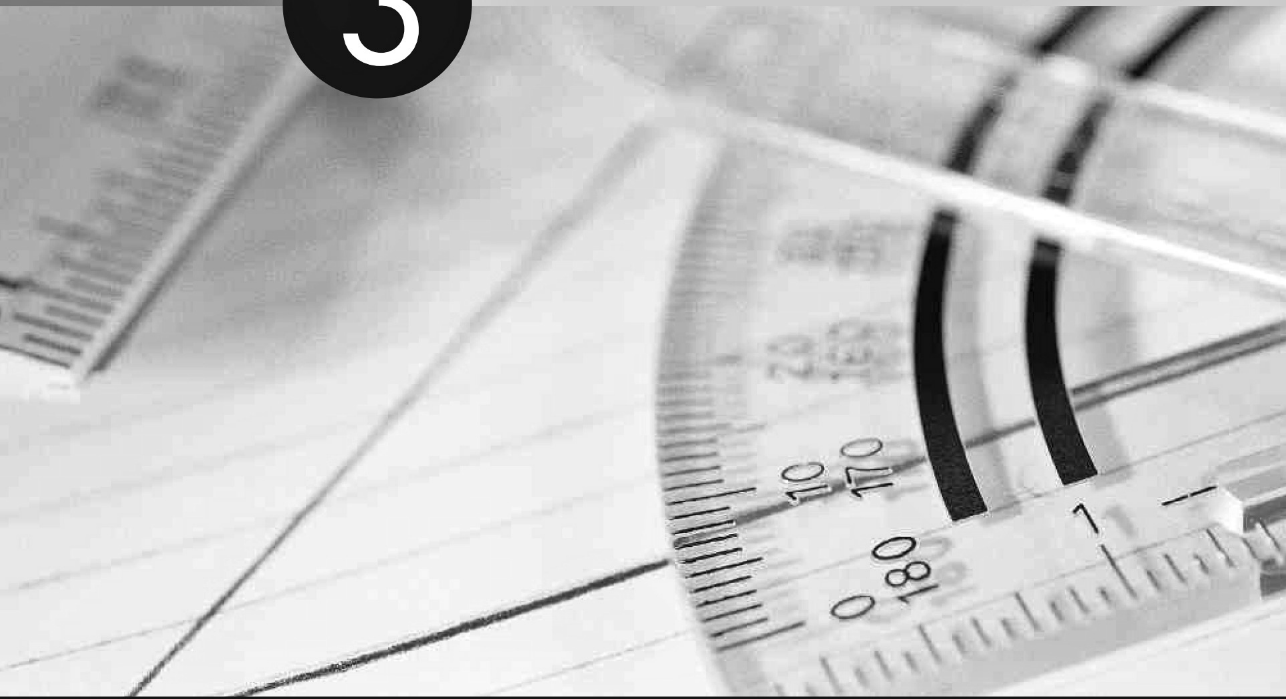
GELSON IEZZI

FUNDAMENTOS DE MATEMÁTICA ELEMENTAR

Trigonometria

3



Atual Editora

RESOLUÇÕES

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3

COMPLEMENTO PARA O PROFESSOR

9ª edição | São Paulo – 2013

Atual Editora

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Apresentação

Este livro é o *Complemento para o Professor* do volume 3, Trigonometria, da coleção *Fundamentos de Matemática Elementar*.

Cada volume desta coleção tem um complemento para o professor, com o objetivo de apresentar a solução dos exercícios mais complicados do livro e sugerir alguns encaminhamentos aos alunos.

É nossa intenção aperfeiçoar continuamente os *Complementos*. Estamos abertos às sugestões e críticas, que nos devem ser encaminhadas por intermédio da Editora.

Agradecemos à professora Erileide Maria de Sobral Souza a colaboração na redação das soluções que são apresentadas neste *Complemento*.

Os Autores.

Sumário

CAPÍTULO II — Razões trigonométricas no triângulo retângulo

6. $\text{tg}\,\hat{B} = \dfrac{b}{c} = \dfrac{\sqrt{5}}{2} \Rightarrow b = \dfrac{c\sqrt{5}}{2}$

$b^2 + c^2 = a^2 \Rightarrow \dfrac{5c^2}{4} + c^2 = 36 \Rightarrow c = 4$ e então $b = 2\sqrt{5}$

14. a) $\text{sen}\ 20°15' = \text{sen}\ 20° + \dfrac{15}{60}(\text{sen}\ 21° - \text{sen}\ 20°) =$

$= 0{,}34202 + \dfrac{15}{60}(0{,}35837 - 0{,}34202) = 0{,}34610$

b) $\cos 15°30' = \cos 15° + \dfrac{30}{60}(\cos 16° - \cos 15°) =$

$= 0{,}96593 + \dfrac{30}{60}(0{,}96126 - 0{,}96593) = 0{,}96358$

c) $\text{tg}\ 12°40' = \text{tg}\ 12° + \dfrac{40}{60}(\text{tg}\ 13° - \text{tg}\ 12°) =$

$= 0{,}21256 + \dfrac{40}{60}(0{,}23087 - 0{,}21256) = 0{,}22476$

d) $\text{sen}\ 50°12' = \text{sen}\ 50° + \dfrac{12}{60}(\text{sen}\ 51° - \text{sen}\ 50°) =$

$= 0{,}76604 + \dfrac{12}{60}(0{,}77715 - 0{,}76604) = 0{,}76826$

e) $\cos 70°27' = \cos 70° + \dfrac{27}{60}(\cos 71° - \cos 70°) =$

$= 0{,}34202 + \dfrac{27}{60}(0{,}32557 - 0{,}34202) = 0{,}33462$

f) $\text{tg}\ 80°35' = \text{tg}\ 80° + \dfrac{35}{60}(\text{tg}\ 81° - \text{tg}\ 80°) =$

$= 5{,}67128 + \dfrac{35}{60}(6{,}31375 - 5{,}67128) = 6{,}04605$

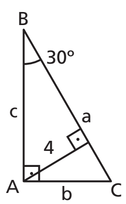
15. $b = 4 \cdot \text{tg}\ 35° \Rightarrow b = 2{,}80084$ cm

$a = \dfrac{4}{\cos 35°} \Rightarrow a = 4{,}88311$ cm

16. $c \cdot \text{sen}\ 30° = 4 \Rightarrow c = 8$

$b = c \cdot \text{tg}\ 30° \Rightarrow b = \dfrac{8\sqrt{3}}{3}$

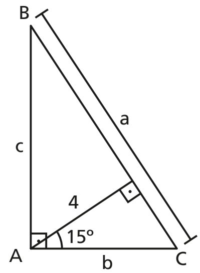
$a^2 = b^2 + c^2 \Rightarrow a = \dfrac{16\sqrt{3}}{3}$



17. $b \cdot \cos 15° = 4 \Rightarrow b = 4\sqrt{2}\left(\sqrt{3} - 1\right)$

$c \cdot \cos 75° = 4 \Rightarrow c = 4\sqrt{2}\left(\sqrt{3} + 1\right)$

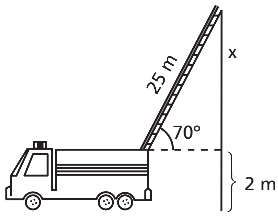
$a^2 = b^2 + c^2 \Rightarrow a = 16$



18. $h = y \cdot \text{tg } 30°$ e $h = x \cdot \text{tg } 60° \Rightarrow \dfrac{x}{y} = \dfrac{1}{3}$

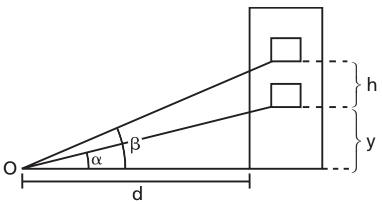
19. $x = 25 \cdot \text{sen } 70° \Rightarrow x = 23{,}49$ m

$x + 2 = 25{,}49$ m



21. $d = \dfrac{h + y}{\text{tg } \beta}$ e $d = \dfrac{y}{\text{tg } \alpha} \Rightarrow$

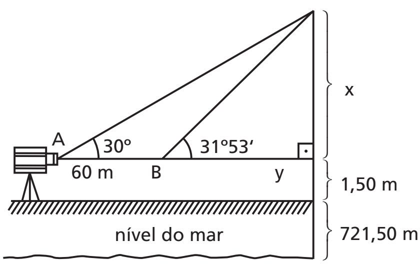
$\Rightarrow h = d\,(\text{tg } \beta - \text{tg } \alpha)$



22. $AB = \dfrac{H - h}{\text{tg } \beta}$ e $AB = \dfrac{h}{\text{tg } \alpha} \Rightarrow H = h\left(\dfrac{\text{tg } \beta}{\text{tg } \alpha} + 1\right)$

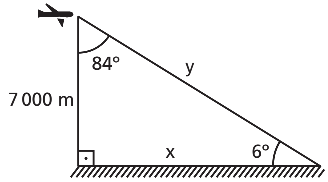
23. $x = y \cdot \text{tg } 31°53'$ e $x = (60 + y) \cdot \text{tg } 30° \Rightarrow x = 503{,}57$ m

$503{,}57 + 1{,}50 + 721{,}50 = 1\,226{,}57$ m

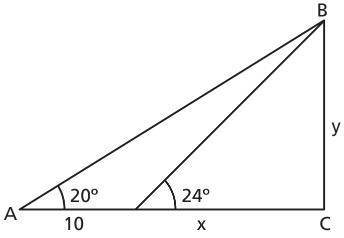


24. $x = 7\,000 \cdot \text{tg } 84° \Rightarrow x = 66{,}60 \text{ km}$

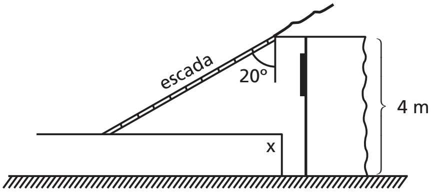
$y = \dfrac{7\,000}{\cos 84°} \Rightarrow y = 66{,}97 \text{ km}$



25. $y = x \cdot \text{tg } 24° \text{ e } y = (10 + x) \cdot \text{tg } 20° \Rightarrow x = 44{,}72\text{m}$



26. $4 - x = 3 \cdot \cos 20° \Rightarrow x = 1{,}18 \text{ m}$



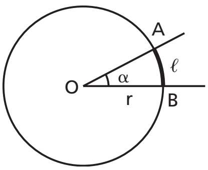
CAPÍTULO III — Arcos e ângulos

35. $\left.\begin{array}{l} a - b = \dfrac{\pi}{12} \\ a + b = \dfrac{7\pi}{4} \end{array}\right\} \Rightarrow a = \dfrac{165\pi}{180} = \dfrac{11\pi}{12} \text{ rad, } b = \dfrac{150\pi}{180} = \dfrac{5\pi}{6} \text{ rad}$

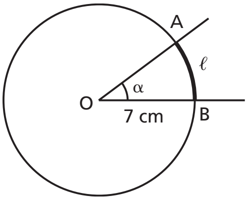
36. $\left.\begin{array}{l} a + b + c = 13° \\ a + b + 2c = 15° \\ a + 2b + c = 20° \end{array}\right\} \Rightarrow a = 4°, b = 7° \text{ e } c = 2°$

39. $\ell = \dfrac{2\pi r}{3}$

$\alpha = \dfrac{\ell}{r} \text{ rad} \Rightarrow \alpha = \dfrac{2\pi}{3} \text{ rad ou } 120°$

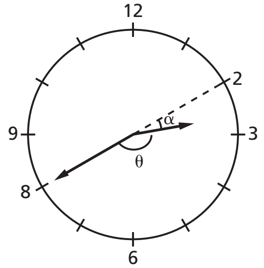


40. $\ell = \alpha \cdot r = 4{,}5 \cdot 7 = 31{,}5$ cm



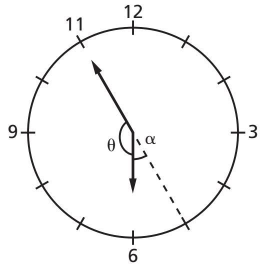
42. a) $\alpha = \dfrac{40}{60} \cdot 30° = 20°$

$\theta = 180° - 20° = 160°$



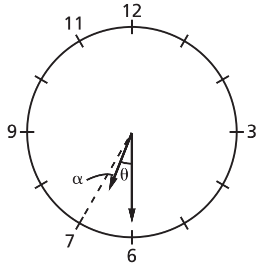
b) $\alpha = \dfrac{55}{60} \cdot 30° = 27{,}5°$

$\theta = 180° - 27{,}5° = 152{,}5°$ ou $152°30'$



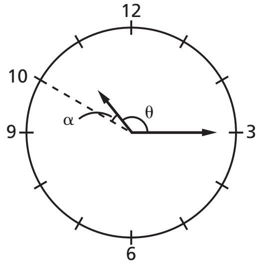
c) $\alpha = \dfrac{30 \cdot 30°}{60} = 15°$

$\theta = 30° - 15° = 15°$



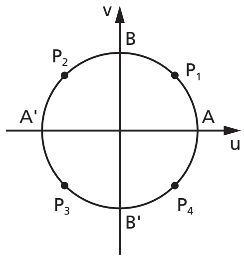
d) $\alpha = \dfrac{15}{60} \cdot 30° = 7{,}5°$

$\theta = 150° - 7{,}5° = 142{,}5°$ ou $142°30'$



44. $\widehat{AP_1} = \frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$ rad

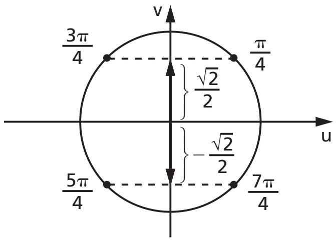
Imagens de *x*	A	P_1	B	P_2	A'	P_3	B'	P_4
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$



CAPÍTULO IV — Razões trigonométricas na circunferência

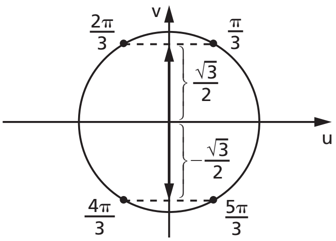
49. $\text{sen}\,\frac{\pi}{4} = \text{sen}\,\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\text{sen}\,\frac{5\pi}{4} = \text{sen}\,\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$



51. $\text{sen}\,\frac{\pi}{3} = \text{sen}\,\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\text{sen}\,\frac{4\pi}{3} = \text{sen}\,\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$



52. a) $\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{3} + \sqrt{2}}{2}$

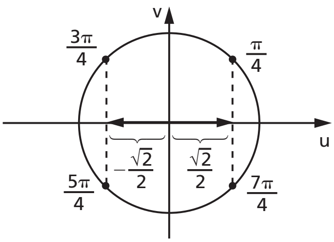
b) $2 \cdot \frac{1}{2} + \frac{1}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) = \frac{4 - \sqrt{2}}{4}$

c) $3(1) - 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2} \cdot 0 = 3 + \sqrt{2}$

d) $-\frac{2}{3}(-1) + \frac{3}{5} \cdot \left(-\frac{\sqrt{3}}{2}\right) - \frac{6}{7}\left(-\frac{1}{2}\right) = \frac{230 - 63\sqrt{3}}{210}$

57. $\cos\frac{\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$

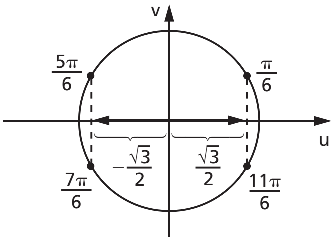
$\cos\frac{3\pi}{4} = \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$



58. $\cos\frac{\pi}{6} = \cos\frac{11\pi}{6} = \frac{\sqrt{3}}{2}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\cos\frac{7\pi}{6} = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$



60. a) $\frac{1}{2} + \frac{\sqrt{2}}{2} - 1 = \frac{-1 + \sqrt{2}}{2}$

b) $2 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{4\sqrt{3} + \sqrt{2}}{4}$

c) $3 \cdot 0 - 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2}(-1) = \frac{2\sqrt{2} - 1}{2}$

d) $-\frac{2}{3} \cdot 0 + \frac{3}{5}\left(\frac{1}{2}\right) - \frac{6}{7}\left(-\frac{\sqrt{3}}{2}\right) = \frac{21 + 30\sqrt{3}}{70}$

63. $0° < 45° < 90° \Rightarrow (\text{sen } 45° > 0 \text{ e } \cos 45° > 0) \Rightarrow y_1 > 0$

$180° < 225° < 270° \Rightarrow (\text{sen } 225° < 0 \text{ e } \cos 225° < 0) \Rightarrow y_2 < 0$

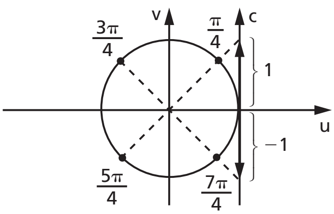
$\frac{3\pi}{2} < \frac{7\pi}{4} < 2\pi \Rightarrow \left(\text{sen}\frac{7\pi}{4} < 0 \text{ e } \cos\frac{7\pi}{4} > 0 \text{ e } \left|\text{sen}\frac{7\pi}{4}\right| = \left|\cos\frac{7\pi}{4}\right|\right) \Rightarrow$

$\Rightarrow y_3 = 0$

$270° < 300° < 315° < 360° \Rightarrow \big(|\text{sen } 300°| > |\cos 300°|;$
$\text{sen } 300° < 0 \text{ e } \cos 300° > 0\big) \Rightarrow y_4 < 0$

66. $\text{tg}\frac{\pi}{4} = \text{tg}\frac{5\pi}{4} = 1$

$\text{tg}\frac{3\pi}{4} = \text{tg}\frac{7\pi}{4} = -1$



69. a) $\sqrt{3} + 1 - 0 = 1 + \sqrt{3}$

b) $2 \cdot \dfrac{\sqrt{3}}{3} + \dfrac{1}{2}(-1) = \dfrac{4\sqrt{3} - 3}{6}$

c) $-2(1) + \dfrac{1}{2} \cdot (0) - \dfrac{1}{3}\left(-\dfrac{\sqrt{3}}{3}\right) = \dfrac{-18 + \sqrt{3}}{9}$

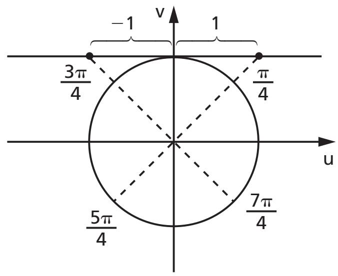
d) $\dfrac{3}{5}(-\sqrt{3}) - \dfrac{6}{7}\left(\dfrac{\sqrt{3}}{3}\right) - \dfrac{2}{3}(0) = \dfrac{-31\sqrt{3}}{35}$

71. a) $\left.\begin{array}{l} 180° < 269° < 270° \Rightarrow \text{tg } 269° > 0 \\ 90° < 178° < 180° \Rightarrow \text{sen } 178° > 0 \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} 0 < \dfrac{5\pi}{11} < \dfrac{\pi}{2} \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \text{tg } \dfrac{12\pi}{7} < 0 \end{array}\right\} \Rightarrow y_2 < 0$

74. $\text{cotg } \dfrac{\pi}{4} = \text{cotg } \dfrac{5\pi}{4} = 1$

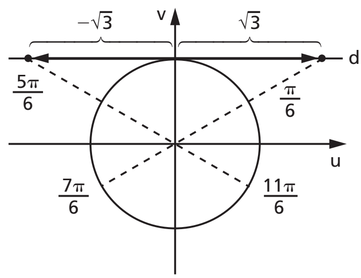
$\text{cotg } \dfrac{3\pi}{4} = \text{cotg } \dfrac{7\pi}{4} = -1$



75. $\text{cotg } \dfrac{\pi}{6} = \text{cotg } \dfrac{7\pi}{6} = \sqrt{3}$

$\text{cotg } \dfrac{5\pi}{6} = -\text{cotg } \dfrac{\pi}{6} = -\sqrt{3}$

$\text{cotg } \dfrac{5\pi}{6} = \text{cotg } \dfrac{11\pi}{6} = -\sqrt{3}$



77. a) $\dfrac{\sqrt{3}}{3} + 1 + \sqrt{3} = \dfrac{3 + 4\sqrt{3}}{3}$

b) $2\left(\dfrac{-\sqrt{3}}{3}\right) - \dfrac{1}{2}(-\sqrt{3}) = -\dfrac{\sqrt{3}}{6}$

c) $\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} - (-\sqrt{3}) + \sqrt{3} = \dfrac{5\sqrt{3} + \sqrt{2}}{2}$

d) $\dfrac{3}{5}\left(-\dfrac{\sqrt{3}}{3}\right) - \dfrac{6}{7} \cdot \sqrt{3} - \dfrac{2}{3}(-1) + \dfrac{4}{5} \cdot \left(-\dfrac{\sqrt{2}}{2}\right) = \dfrac{-111\sqrt{3} - 42\sqrt{2} + 70}{105}$

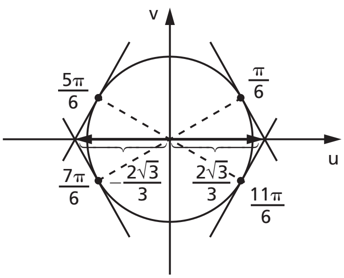
79. a) $\left.\begin{array}{l} 180° < 269° < 270° \Rightarrow \text{cotg } 269° > 0 \\ 90° < 178° < 180° \Rightarrow \text{sen } 178° > 0 \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} \dfrac{\pi}{2} < \dfrac{5\pi}{11} < \pi \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \text{cotg } \dfrac{12\pi}{7} < 0 \end{array}\right\} \Rightarrow y_2 < 0$

81. $\sec \dfrac{5\pi}{6} = -\sec \dfrac{\pi}{6} = -\dfrac{2\sqrt{3}}{3}$

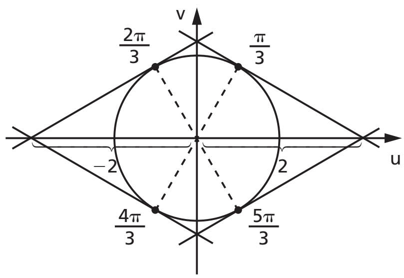
$\sec \dfrac{7\pi}{6} = -\sec \dfrac{\pi}{6} = -\dfrac{2\sqrt{3}}{3}$

$\sec \dfrac{11\pi}{6} = \sec \dfrac{\pi}{6} = \dfrac{2\sqrt{3}}{3}$



82. $\sec \dfrac{5\pi}{3} = \sec \dfrac{\pi}{3} = 2$

$\sec \dfrac{2\pi}{3} = \sec \dfrac{4\pi}{3} = -2$

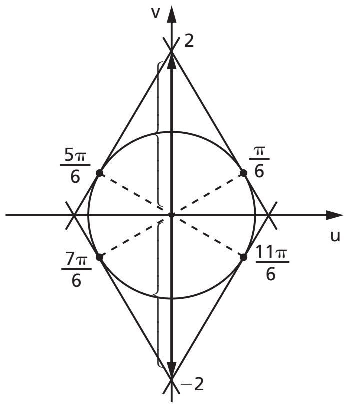


84. a) $\left.\begin{array}{l} 180° < 269° < 270° \Rightarrow \sec 269° < -1 \\ 90° < 178° < 180° \Rightarrow 0 < \text{sen } 178° < 1 \end{array}\right\} \Rightarrow y_1 < 0$

b) $\left.\begin{array}{l} \pi < \dfrac{5\pi}{11} < \dfrac{\pi}{2} \Rightarrow \text{sen } \dfrac{5\pi}{11} > 0 \\ \dfrac{3\pi}{2} < \dfrac{23\pi}{12} < 2\pi \Rightarrow \cos \dfrac{23\pi}{12} > 0 \\ \dfrac{3\pi}{2} < \dfrac{12\pi}{7} < 2\pi \Rightarrow \sec \dfrac{17\pi}{7} > 0 \end{array}\right\} \Rightarrow y_2 > 0$

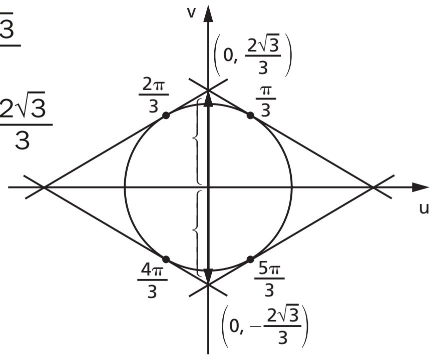
86. $\text{cossec}\ \frac{5\pi}{6} = \text{cossec}\ \frac{\pi}{6} = 2$

$\text{cossec}\ \frac{7\pi}{6} = \text{cossec}\ \frac{11\pi}{6} = -2$



87. $\text{cossec}\ \frac{2\pi}{3} = \text{cossec}\ \frac{\pi}{3} = \frac{2\sqrt{3}}{3}$

$\text{cossec}\ \frac{4\pi}{3} = \text{cossec}\ \frac{5\pi}{3} = -\frac{2\sqrt{3}}{3}$



89. a) $\left.\begin{array}{l} 90° < 91° < 180° \Rightarrow \cos 91° < 0 \text{ e cossec } 91° > 0 \\ |\cos 91°| < |\text{cossec } 91°| \end{array}\right\} \Rightarrow y_1 > 0$

b) $\left.\begin{array}{l} 90° < 107° < 180° \Rightarrow \text{sen } 107° > 0 \text{ e sec } 107° < 0 \\ |\cos 107°| < |\sec 107°| \end{array}\right\} \Rightarrow y_2 < 0$

c) $\left.\begin{array}{l} 0 < \frac{\pi}{7} < \frac{\pi}{2} \Rightarrow \text{cotg}\ \frac{\pi}{7} > 0 \\ \pi < \frac{7\pi}{6} < \frac{3\pi}{2} \Rightarrow \text{tg}\ \frac{7\pi}{6} > 0 \\ \pi < \frac{9\pi}{8} < \frac{3\pi}{2} \Rightarrow \sec \frac{9\pi}{8} < 0 \end{array}\right\} \Rightarrow y_3 < 0$

90. $\left(2 + \frac{1}{2}\right)\left(\frac{\sqrt{2}}{2} - 2\right) = 1{,}25(\sqrt{2} - 4)$

CAPÍTULO V — Relações fundamentais

92. $\text{cossec } x = -\dfrac{25}{24} \Rightarrow \text{sen } x = -\dfrac{24}{25}$

$\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = -\dfrac{7}{25}$

e daí $\text{tg } x = \dfrac{24}{7}$, $\text{cotg } x = \dfrac{7}{24}$ e $\sec x = -\dfrac{25}{7}$

94. $\text{cotg } x = \dfrac{2\sqrt{m}}{m - 1} \Rightarrow \text{tg } x = \dfrac{(m - 1)\sqrt{m}}{2m}$

$\text{tg}^2 x + 1 = \sec^2 x \Rightarrow \sec x = \pm\dfrac{m + 1}{2\sqrt{m}} \Rightarrow \cos x = \pm\dfrac{2\sqrt{m}}{m + 1}$

95. $\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \pm\dfrac{(a^2 - b^2)}{a^2 + b^2} \Rightarrow \sec x = \pm\dfrac{(a^2 + b^2)}{a^2 - b^2}$

97. $\text{sen } x = \dfrac{1}{3} \Rightarrow \text{cossec } x = 3$

$$\left.\begin{array}{l} \text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \dfrac{2\sqrt{2}}{3} \\ \qquad\qquad\qquad\qquad \text{sen } x = \dfrac{1}{3} \end{array}\right\} \Rightarrow \text{cotg } x = 2\sqrt{2}$$

$y = \dfrac{2 \text{ cossec } x}{\text{cossec}^2 x - \text{cotg}^2 x} = \dfrac{2 \cdot 3}{9 - 8} \Rightarrow y = 6$

99. $\cos x = \dfrac{2}{5} \Rightarrow \sec x = \dfrac{5}{2}$; $\sec^2 x = \text{tg}^2 x + 1 \Rightarrow \text{tg}^2 x = \dfrac{21}{4}$

$y = \left(1 + \dfrac{21}{4}\right)^2 + \left(1 - \dfrac{21}{4}\right)^2 \Rightarrow y = \dfrac{457}{8}$

101. $5 \sec x - 3(\sec^2 x - 1) = 1 \Rightarrow 3 \sec^2 x - 5 \sec x - 2 = 0$

e daí $\sec x = -\dfrac{1}{3} \Rightarrow \cos x = -3$ (não convém), $\sec x = 2 \Rightarrow$

$\Rightarrow \cos x = \dfrac{1}{2}$

$\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \text{sen } x = \pm\dfrac{\sqrt{3}}{2}$

102. $\text{sen}^2 x + \cos^2 x - 5 \text{ sen } x \cos x = 3 \Rightarrow -5 \text{ sen } x \cos x = 2 \Rightarrow$

$\Rightarrow \dfrac{-5 \text{ sen } x \cos x}{\cos^2 x} = \dfrac{2}{\cos^2 x} \Rightarrow -5 \text{ tg } x = 2(1 + \text{tg}^2 x) \Rightarrow$

$\Rightarrow 2 \text{ tg}^2 x + 5 \text{ tg } x + 2 = 0 \Rightarrow \text{tg } x = -2$ ou $\text{tg } x = -\dfrac{1}{2}$

104. $\text{cotg}\, x = \dfrac{1}{\text{tg}\, x} \Rightarrow \dfrac{m}{3} = \dfrac{1}{m-2} \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow m = 3$ ou $m = -1$

105. $\text{cossec}\, x = \dfrac{a+1}{\sqrt{a+2}} \Rightarrow \text{sen}\, x = \dfrac{\sqrt{a+2}}{a+1}$

$\text{sen}^2\, x + \cos^2 x = 1 \Rightarrow \left(\dfrac{\sqrt{a+2}}{a+1}\right)^2 + \left(\dfrac{1}{a+1}\right)^2 = 1 \Rightarrow$

$\Rightarrow a^2 + a - 2 = 0 \Rightarrow a = 1$

108. $(\text{sen}\, x + \cos x)^2 = 1 + 2\, \text{sen}\, x \cos x \Rightarrow a^2 = 1 + 2b \Rightarrow a^2 - 2b = 1$

110. $\text{sen}\, x + \cos x = a \Rightarrow \text{sen}^2\, x + 2 \cdot \text{sen}\, x \cdot \cos x + \cos^2 x = a^2 \Rightarrow$

$\Rightarrow \text{sen}\, x \cdot \cos x = \dfrac{a^2 - 1}{2}$

$y = (\text{sen}\, x + \cos x)(\text{sen}^2\, x - \text{sen}\, x \cdot \cos x + \cos^2 x) =$

$= a \cdot \left(1 - \dfrac{a^2-1}{2}\right) = \dfrac{a(3 - a^2)}{2}$

CAPÍTULO VI — Arcos notáveis

112. $\ell_8 = \sqrt{1(2 - \sqrt{4-2})} = \sqrt{2 - \sqrt{2}}$; $\text{sen}\, \dfrac{\pi}{8} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

$\text{sen}^2\, \dfrac{\pi}{8} + \cos^2 \dfrac{\pi}{8} = 1 \Rightarrow \cos \dfrac{\pi}{8} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$; $\text{tg}\, \dfrac{\pi}{8} = \sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}} \Rightarrow$

$\Rightarrow \text{tg}\, \dfrac{\pi}{8} = -1 + \sqrt{2}$

113. $\text{sen}\, \dfrac{\pi}{5} = \dfrac{\sqrt{10 - 2\sqrt{5}}}{4}$ e $\text{sen}^2\, \dfrac{\pi}{5} + \cos^2 \dfrac{\pi}{5} = 1 \Rightarrow \cos \dfrac{\pi}{5} = \dfrac{\sqrt{6 + 2\sqrt{5}}}{4}$

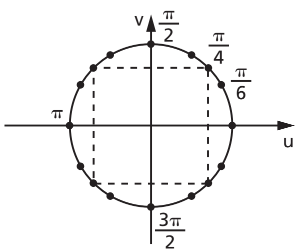
$\text{tg}\, \dfrac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}$; $\text{sen}\, \dfrac{\pi}{10} = \dfrac{\sqrt{5} - 1}{4}$ e $\text{sen}^2\, \dfrac{\pi}{10} + \cos^2 \dfrac{\pi}{10} = 1 \Rightarrow$

$\Rightarrow \cos \dfrac{\pi}{10} = \dfrac{\sqrt{10 + 2\sqrt{5}}}{4}$; $\text{tg}\, \dfrac{\pi}{10} = \dfrac{\sqrt{5} - 1}{\sqrt{10 + 2\sqrt{5}}} = \dfrac{\sqrt{25 - 10\sqrt{5}}}{5}$

115. $A = \left\{0, \dfrac{1}{2}, \dfrac{\sqrt{3}}{2}, 1, -\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}, -1\right\}$

$B = \left\{1, \dfrac{\sqrt{2}}{2}, 0, -\dfrac{\sqrt{2}}{2}, -1\right\}$

$A \cap B = \{-1, 0, 1\}$



CAPÍTULO VII — Redução ao 1º quadrante

118. $\text{sen}\left(x + \frac{\pi}{2}\right) = \text{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \text{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{3}{5}$

119. a) $\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

b) $\cos\left(x + \frac{\pi}{2}\right) = -\cos\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = -\cos\left(\frac{\pi}{2} - x\right) =$

$= -\text{sen}\ x = -\frac{1}{2}$

c) $\text{sen}\left(x + \frac{\pi}{2}\right) = \text{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \text{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{\sqrt{3}}{2}$

d) $\text{tg}\left(x + \frac{\pi}{2}\right) = \dfrac{\text{sen}\left(x + \frac{\pi}{2}\right)}{\cos\left(x + \frac{\pi}{2}\right)} = -\sqrt{3}$

e) $\text{cotg}\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\text{tg}\left(x + \frac{\pi}{2}\right)} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

f) $\sec\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\cos\left(x + \frac{\pi}{2}\right)} = -2$

g) $\text{cossec}\left(x + \frac{\pi}{2}\right) = \dfrac{1}{\text{sen}\left(x + \frac{\pi}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

120. a) $[\text{sen}\ x + \text{sen}\ x][\text{cotg}\ x + \text{cotg}\ x] = 2\ \text{sen}\ x \cdot 2\ \text{cotg}\ x =$

$= 4\ \text{sen}\ x \cdot \dfrac{\cos x}{\text{sen}\ x} = 4 \cos x$

b) $\dfrac{\text{tg}\ x - \sec x}{[\text{tg}\ x + \text{cossec}\ x] \cdot \text{sen}\ x} = \dfrac{\dfrac{\text{sen}\ x - 1}{\cos x}}{\left(\dfrac{\text{sen}^2 x + \cos x}{\cos x \cdot \text{sen}\ x}\right) \cdot \text{sen}\ x} =$

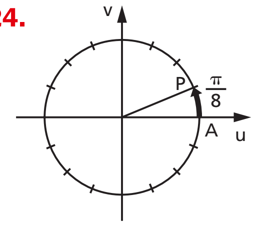
$= \dfrac{\text{sen}\ x - 1}{\text{sen}^2 x + \cos x}$

121. $\dfrac{\text{sen}\ x - \text{sen}\ x + \text{tg}\ x}{-\text{tg}\ x - \cos x + \cos x} = \dfrac{\text{tg}\ x}{-\text{tg}\ x} = -1$

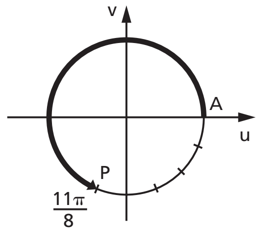
122. $\dfrac{-\text{sen}\ x - \cos x + \cos x + 3\ \text{sen}\ x}{-\cos x - \cos x - \text{sen}\ x + \text{sen}\ x} = \dfrac{2\ \text{sen}\ x}{-2 \cos x} = -\text{tg}\ x$

CAPÍTULO VIII — Funções circulares

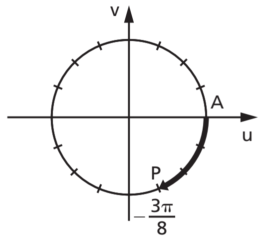
124.



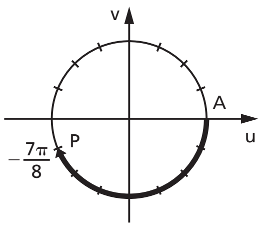
$\widehat{AP} = \dfrac{1}{16}$ do ciclo, no sentido anti-horário



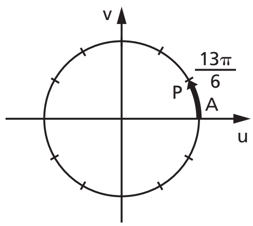
$\widehat{AP} = \dfrac{11}{16}$ do ciclo, no sentido anti-horário



$\widehat{AP} = \dfrac{3}{16}$ do ciclo, no sentido horário

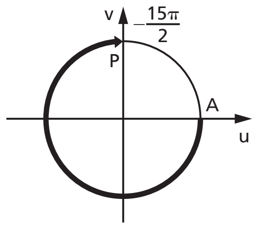


$\widehat{AP} = \dfrac{7}{16}$ do ciclo, no sentido horário



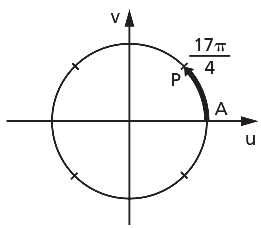
$\dfrac{13\pi}{6}$ e $\dfrac{\pi}{6}$ são côngruos

$\widehat{AP} = \dfrac{1}{12}$ do ciclo, no sentido anti-horário



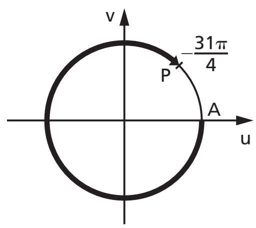
$-\dfrac{15\pi}{2}$ e $-\dfrac{3\pi}{2}$ são côngruos

$\widehat{AP} = \dfrac{3}{4}$ do ciclo, no sentido horário



$\dfrac{17\pi}{4}$ e $\dfrac{\pi}{4}$ são côngruos

$\widehat{AP}$ é $\dfrac{1}{8}$ do ciclo, no sentido anti-horário

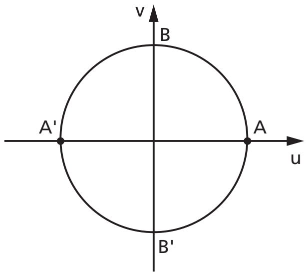


$-\dfrac{31\pi}{4}$ e $-\dfrac{7\pi}{4}$ são côngruos

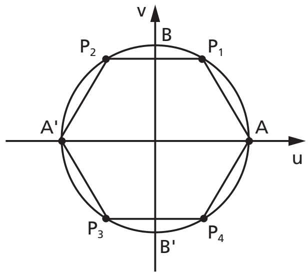
$\widehat{AP}$ é $\dfrac{7}{8}$ do ciclo, no sentido horário

126.

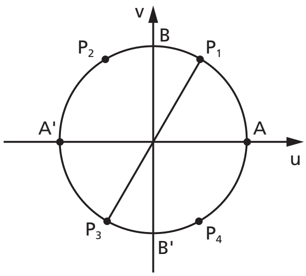
$E = \{A', A\}$



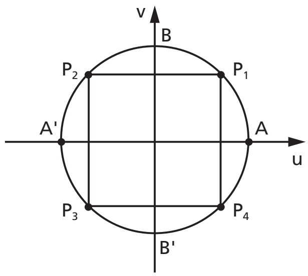
$F = \{A, P_1, P_2, A', P_3, P_4\}$



$G = \{P_1, P_3\}$



$H = \{P_1, P_2, P_3, P_4\}$



127.

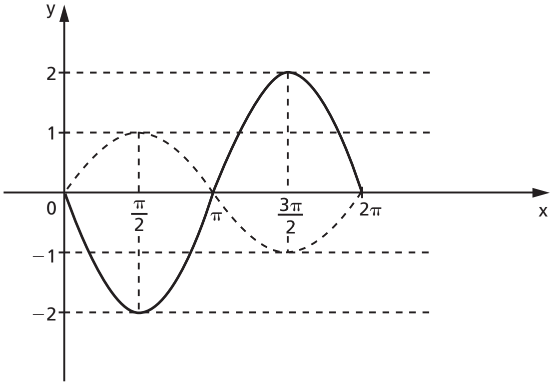
a) $830° = 2(360°) + 110° \Rightarrow \text{sen } 830° = \text{sen } 110° = \text{sen } 70°$
$1\,195° = 3(360°) + 115° \Rightarrow \text{sen } 1\,195° = \text{sen } 115° = \text{sen } 65°$
$0° < x < 90° \Rightarrow \text{sen } x$ é crescente $\Rightarrow \text{sen } 830° > \text{sen } 1\,195°$

b) $-535° = -360° - 175° \Rightarrow \cos(-535°) = \cos(-175°) = -\cos 5°$
$\cos 190° = -\cos 10°$; $0° < x < 90° \Rightarrow \cos x$ é decrescente $\Rightarrow$
$\Rightarrow \cos 5° > \cos 10° \Rightarrow \cos 190° > \cos(-535°)$

130.

x	sen x	y = −2 sen x
0	0	0
$\frac{\pi}{2}$	1	−2
π	0	0
$\frac{3\pi}{2}$	−1	2
2π	0	0

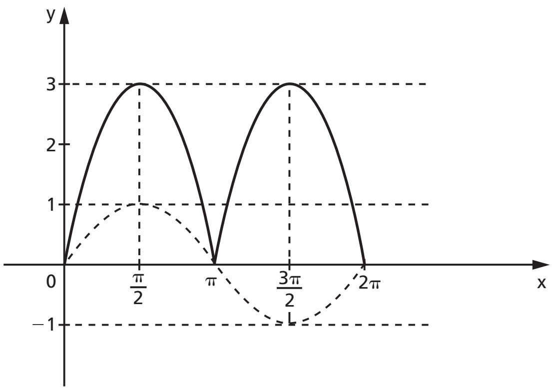
$\text{Im}(f) = [-2, 2]$
$p = 2\pi$



132.

x	sen x	y = |3 sen x|
0	0	0
$\frac{\pi}{2}$	1	3
π	0	0
$\frac{3\pi}{2}$	−1	3
2π	0	0

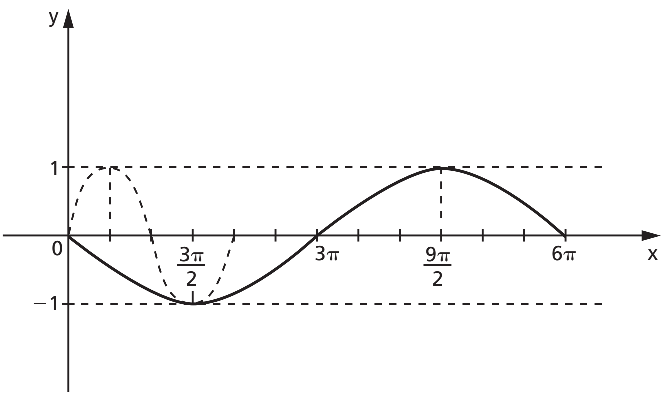
$Im(f) = [0, 3]$
$p = \pi$



136.

x	$t = \frac{x}{3}$	$y = -\text{sen } t$
0	0	0
$\frac{3\pi}{2}$	$\frac{\pi}{2}$	-1
3π	π	0
$\frac{9\pi}{2}$	$\frac{3\pi}{2}$	1
6π	2π	0

$\text{Im}(f) = [-1, 1]$
$p = 6\pi$

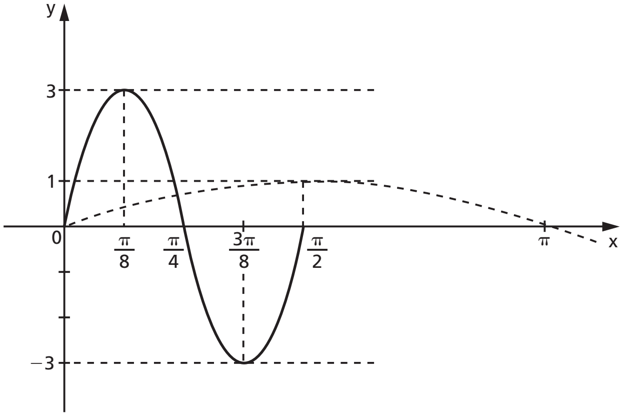


137.

x	$t = 4x$	$y = 3 \text{ sen } t$
0	0	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	3
$\frac{\pi}{4}$	π	0
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	-3
$\frac{\pi}{2}$	2π	0

$\text{Im}(f) = [-3, 3]$

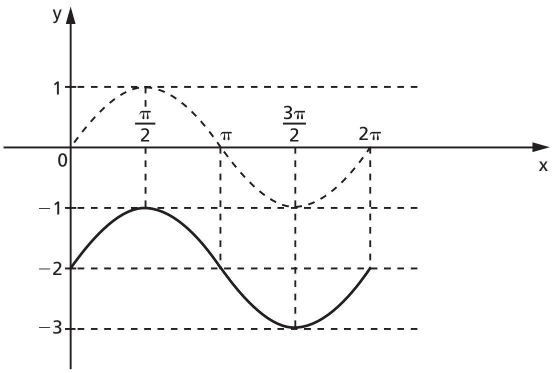
$p = \frac{\pi}{2}$



139.

x	sen x	y = −2 + sen x
0	0	−2
$\frac{\pi}{2}$	1	−1
π	0	−2
$\frac{3\pi}{2}$	−1	−3
2π	0	−2

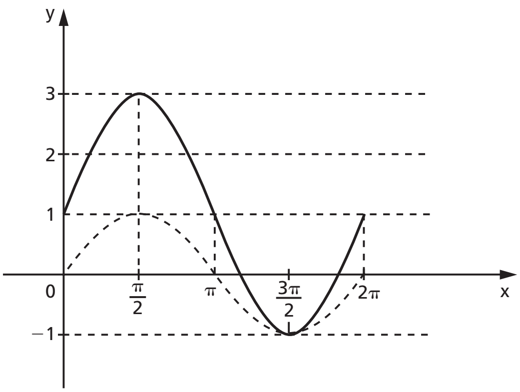
$Im(f) = [-3, -1]$
$p = 2\pi$



140.

x	sen x	$y = 1 + 2$ sen x
0	0	1
$\frac{\pi}{2}$	1	3
π	0	1
$\frac{3\pi}{2}$	-1	-1
2π	0	1

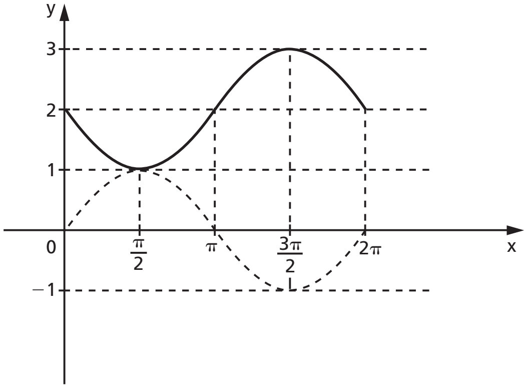
$\text{Im}(f) = [-1, 3]$
$p = 2\pi$



141.

x	sen x	$y = 2 -$ sen x
0	0	2
$\frac{\pi}{2}$	1	1
π	0	2
$\frac{3\pi}{2}$	-1	3
2π	0	2

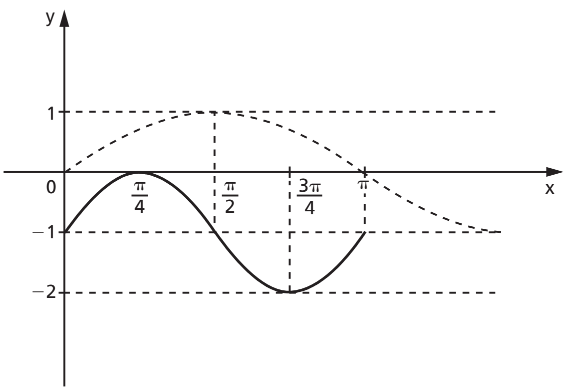
$\text{Im}(f) = [1, 3]$
$p = 2\pi$



142.

x	t = 2x	sen t	y = −1 + sen t
0	0	0	−1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1	0
$\frac{\pi}{2}$	π	0	−1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	−1	−2
π	2π	0	−1

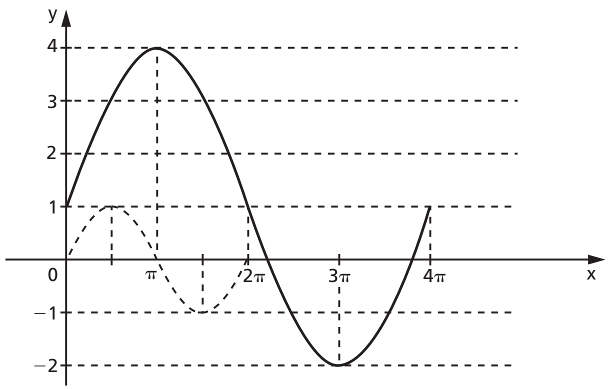
$Im(f) = [-2, 0]$
$p = \pi$



143.

x	$t = \frac{x}{2}$	sen t	y = 1 + 3 sen t
0	0	0	1
π	$\frac{\pi}{2}$	1	4
2π	π	0	1
3π	$\frac{3\pi}{2}$	-1	-2
4π	2π	0	1

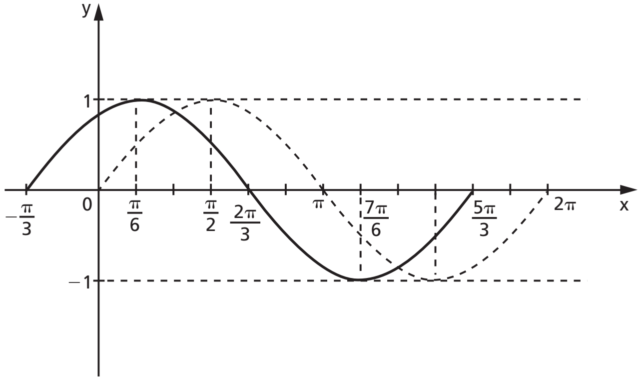
$Im(f) = [-2, 4]$
$p = 4\pi$



145.

x	$t = x + \frac{\pi}{3}$	y = sen t
$-\frac{\pi}{3}$	0	0
$\frac{\pi}{6}$	$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	π	0
$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	2π	0

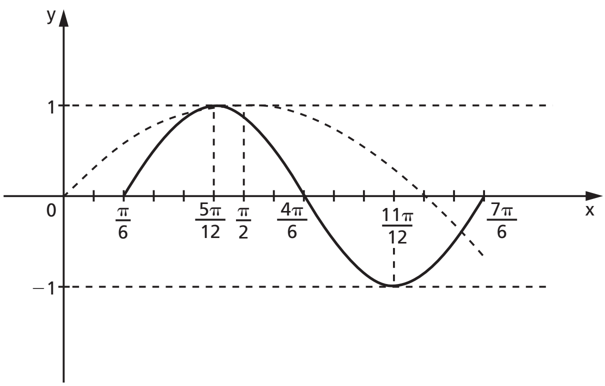
$Im(f) = [-1, 1]$
$p = 2\pi$



146.

x	$t = 2x - \frac{\pi}{3}$	y = sen t
$\frac{\pi}{6}$	0	0
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1
$\frac{4\pi}{6}$	π	0
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	−1
$\frac{7\pi}{6}$	2π	0

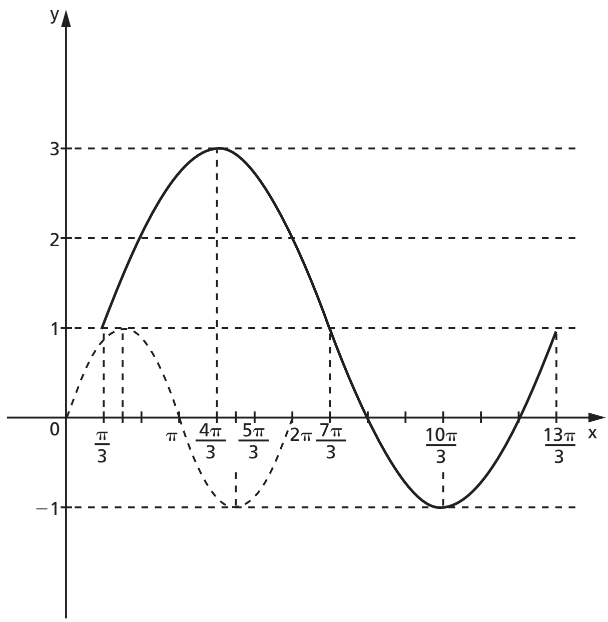
$\text{Im}(f) = [-1, 1]$
$p = \pi$



147.

x	$t = \frac{x}{2} - \frac{\pi}{6}$	sen t	$y = 1 + 2$ sen t
$\frac{\pi}{3}$	0	0	1
$\frac{4\pi}{3}$	$\frac{\pi}{2}$	1	3
$\frac{7\pi}{3}$	π	0	1
$\frac{10\pi}{3}$	$\frac{3\pi}{2}$	-1	-1
$\frac{13\pi}{3}$	2π	0	1

$Im(f) = [-1, 3]$
$p = 4\pi$

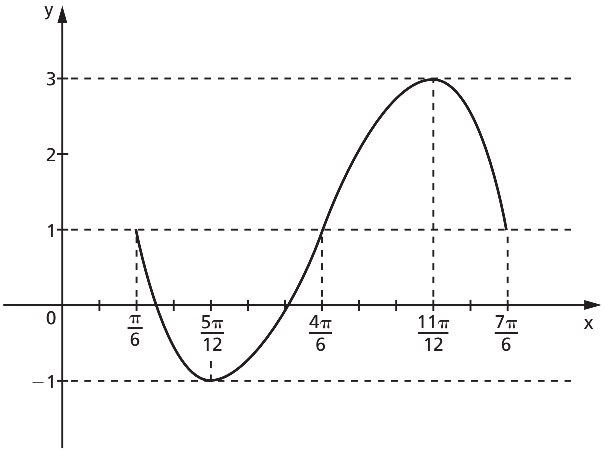


149. $t = 2\pi x + \frac{\pi}{2} \Rightarrow p = \frac{2\pi}{c} = \frac{2\pi}{2\pi} \Rightarrow p = 1$

150.

x	$t = 2x - \frac{\pi}{3}$	sen t	$y = 1 - 2$ sen t
$\frac{\pi}{6}$	0	0	1
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1	-1
$\frac{4\pi}{6}$	π	0	1
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	-1	3
$\frac{7\pi}{6}$	2π	0	1

$\text{Im}(f) = [-1, 3]$
$p = \pi$



152. a) $-1 \leqslant 2 - 5m \leqslant 1 \Rightarrow -3 \leqslant -5m \leqslant -1 \Rightarrow \frac{1}{5} \leqslant m \leqslant \frac{3}{5}$

b) $\frac{m-1}{m-2} \geqslant -1 \Rightarrow \frac{2m-3}{m-2} \geqslant 0 \Rightarrow m \leqslant \frac{3}{2}$ ou $m > 2$ (A)

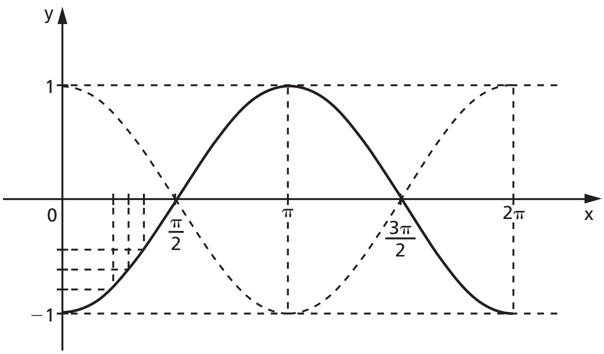
$\frac{m-1}{m-2} \leqslant 1 \Rightarrow \frac{1}{m-2} \leqslant 0 \Rightarrow m < 2$ (B)

Fazendo a interseção de (A) com (B), vem $m \leqslant \frac{3}{2}$.

153.

x	cos x	y = − cos x
0	1	−1
$\frac{\pi}{2}$	0	0
π	−1	1
$\frac{3\pi}{2}$	0	0
2π	1	−1

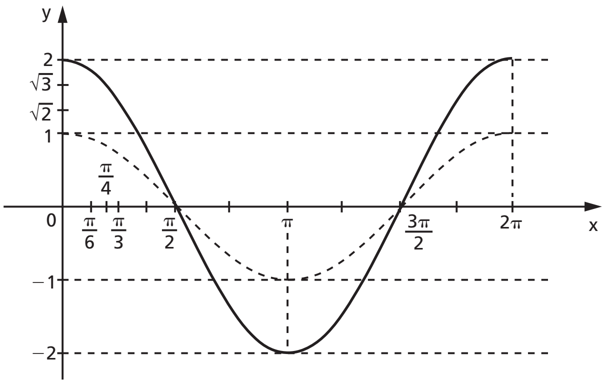
$Im(f) = [-1, 1]$
$p = 2\pi$



154.

x	cos x	y = 2 cos x
0	1	2
$\frac{\pi}{2}$	0	0
π	−1	−2
$\frac{3\pi}{2}$	0	0
2π	1	2

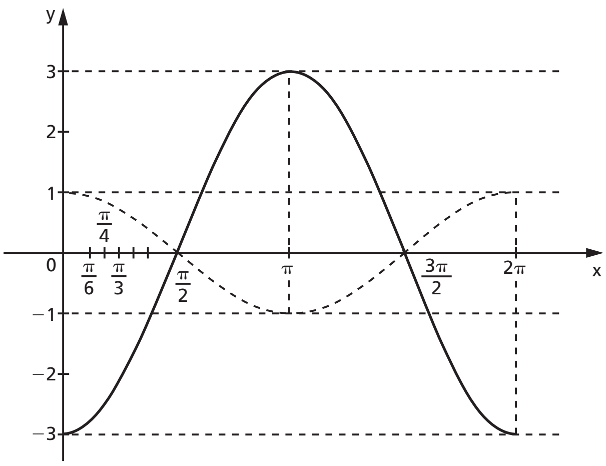
$Im(f) = [-2, 2]$
$p = 2\pi$



155.

x	cos x	y = −3 cos x
0	1	−3
$\frac{\pi}{2}$	0	0
π	−1	3
$\frac{3\pi}{2}$	0	0
2π	1	−3

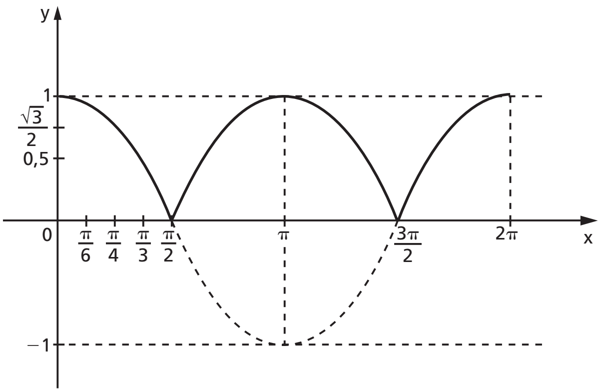
$Im(f) = [-3, 3]$
$p = 2\pi$



156.

x	$\cos x$	$y = \|\cos x\|$
0	1	1
$\frac{\pi}{2}$	0	0
π	-1	1
$\frac{3\pi}{2}$	0	0
2π	1	1

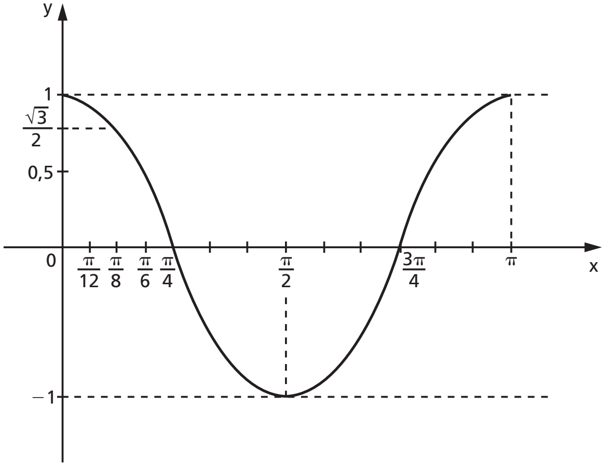
$Im(f) = [0, 1]$
$p = \pi$



157.

x	$t = 2x$	$y = \cos t$
0	0	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1

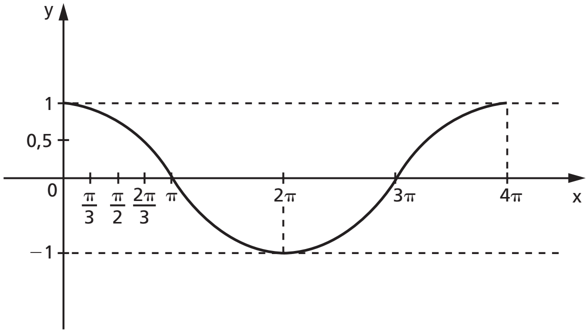
$Im(f) = [-1, 1]$
$p = \pi$



158.

x	$t = \frac{x}{2}$	$y = \cos t$
0	0	1
π	$\frac{\pi}{2}$	0
2π	π	-1
3π	$\frac{3\pi}{2}$	0
4π	2π	1

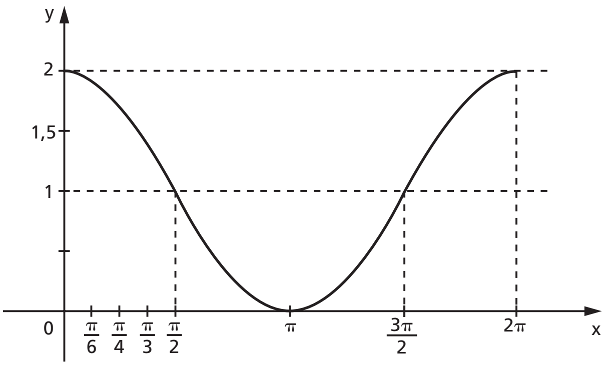
$Im(f) = [-1, 1]$
$p = 4\pi$



159.

x	$\cos x$	$y = 1 + \cos x$
0	1	2
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	1
2π	1	2

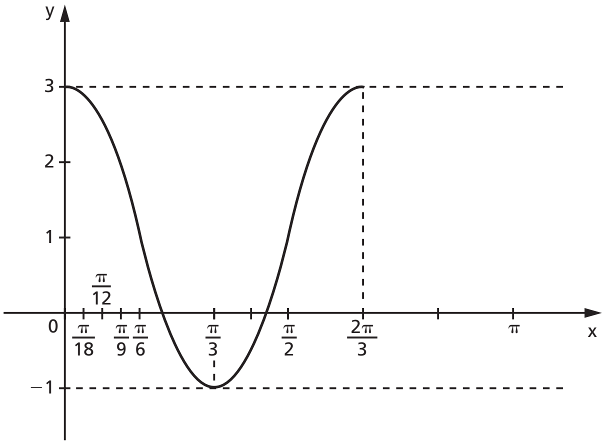
$\text{Im}(f) = [0, 2]$
$p = 2\pi$



160.

x	$t = 3x$	$\cos t$	$y = 1 + 2\cos t$
0	0	1	3
$\frac{\pi}{6}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{3}$	π	-1	-1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	0	1
$\frac{2\pi}{3}$	2π	1	3

$\text{Im}(f) = [-1, 3]$
$p = \frac{2\pi}{3}$

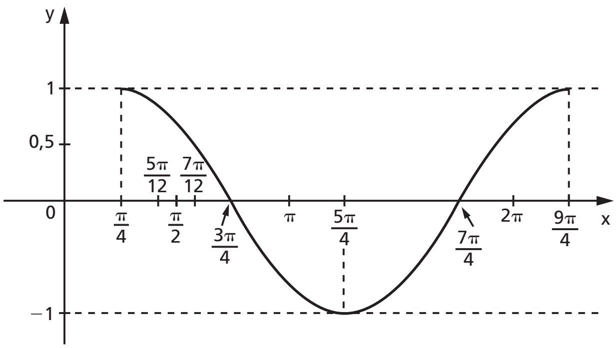


161.

x	$t = x - \frac{\pi}{4}$	$y = \cos t$
$\frac{\pi}{4}$	0	1
$\frac{3\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{5\pi}{4}$	π	-1
$\frac{7\pi}{4}$	$\frac{3\pi}{2}$	0
$\frac{9\pi}{4}$	2π	1

$Im(f) = [-1, 1]$

$p = \frac{9\pi}{4} - \frac{\pi}{4} = 2\pi$

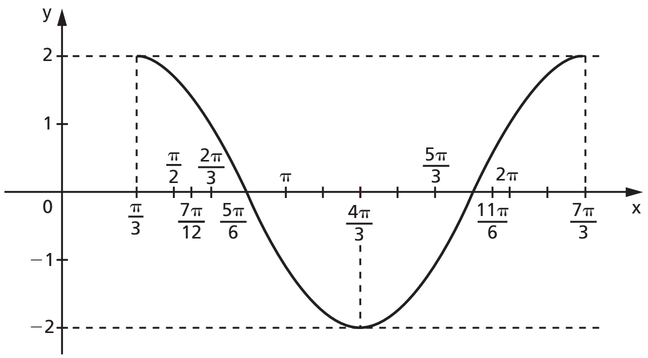


162.

x	$t = x - \frac{\pi}{3}$	$y = 2\cos t$
$\frac{\pi}{3}$	0	2
$\frac{5\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{4\pi}{3}$	π	-2
$\frac{11\pi}{6}$	$\frac{3\pi}{2}$	0
$\frac{7\pi}{3}$	2π	2

$\text{Im}(f) = [-2, 2]$

$p = \frac{7\pi}{3} - \frac{\pi}{3} = 2\pi$

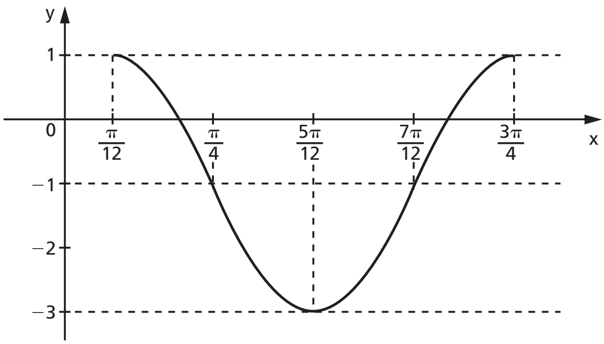


163.

x	$t = 3x - \frac{\pi}{4}$	$\cos t$	$y = -1 + 2\cos t$
$\frac{\pi}{12}$	0	1	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0	-1
$\frac{5\pi}{12}$	π	-1	-3
$\frac{7\pi}{12}$	$\frac{3\pi}{2}$	0	-1
$\frac{3\pi}{4}$	2π	1	1

$\text{Im}(f) = [-3, 1]$

$p = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$



164. $\frac{t+2}{2t-1} \geqslant -1 \Rightarrow \frac{3t+1}{2t-1} \geqslant 0 \Rightarrow t \leqslant -\frac{1}{3}$ ou $t > \frac{1}{2}$ (A)

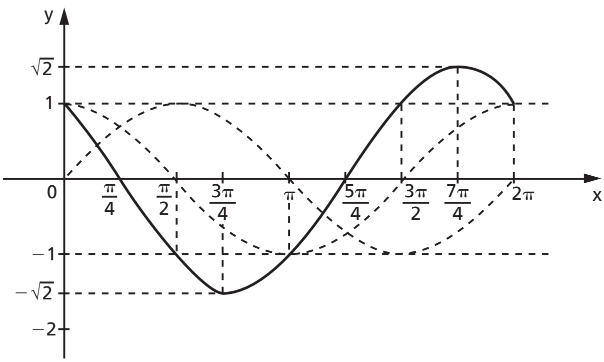
$\frac{t+2}{2t-1} \leqslant 1 \Rightarrow \frac{-t+3}{2t-1} \leqslant 0 \Rightarrow t < \frac{1}{2}$ ou $t \geqslant 3$ (B)

Fazendo a interseção de (A) com (B), vem $t \leqslant -\frac{1}{3}$ ou $t \geqslant 3$.

166.

x	cos x	sen x	y = cos x − sen x
0	1	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0
$\frac{\pi}{2}$	0	1	-1
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$-\sqrt{2}$
π	-1	0	-1
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	0
$\frac{3\pi}{2}$	0	-1	1
$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\sqrt{2}$
2π	1	0	1

$p = 2\pi$



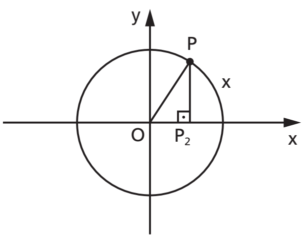
167. **Solução 1**

$0 < x < \frac{\pi}{2} \Rightarrow \text{sen } x > 0 \text{ e sen } x < 1 \Rightarrow \text{sen}^2 x < \text{sen } x \ \ (1)$

$0 < x < \frac{\pi}{2} \Rightarrow \cos x > 0 \text{ e } \cos x < 1 \Rightarrow \cos^2 x < \cos x \ \ (2)$

De (1) + (2) $\Rightarrow \text{sen } x + \cos x > 1$.

Solução 2



No triângulo OP_2P temos:

$\overline{OP_2} + \overline{P_2P} > \overline{OP}$

(um lado é sempre menor que a soma dos outros dois)

Então:

$\cos x + \text{sen } x > 1$

168. $t = 4x$; $t = 0 \Rightarrow x = 0$; $t = 2\pi \Rightarrow x = \frac{\pi}{2}$; $p = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

169. A sequência é $\cos \alpha, -\cos \alpha, \cos \alpha, -\cos \alpha, \ldots$; então: a soma dos 12 termos iniciais é zero.

171. a) $t = 3x$; $3x \neq \frac{\pi}{2} + k\pi$; $D(f) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k\frac{\pi}{3}, k \in \mathbb{Z}\right\}$

b) $t = 2x - \frac{\pi}{3}$; $2x - \frac{\pi}{3} \neq \frac{\pi}{2} + k\pi$; $D(f) = \left\{x \in \mathbb{R} \mid x \neq \frac{5\pi}{12} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

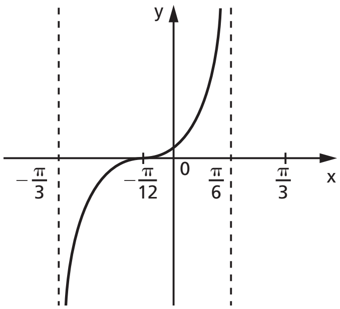
172. $\alpha^2 - 5\alpha + 4 \geqslant 0 \Rightarrow \alpha \leqslant 1 \text{ ou } \alpha \geqslant 4$

174. $t = 2x + \frac{\pi}{6}$; $2x + \frac{\pi}{6} \neq \frac{\pi}{2} + k\pi$;

$D(f) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

$-\frac{\pi}{2} < t < \frac{\pi}{2} \Rightarrow -\frac{\pi}{3} < x < \frac{\pi}{6}$;

$p = \frac{\pi}{6} - \left(-\frac{\pi}{3}\right) = \frac{\pi}{2}$



175. $t = x - \frac{\pi}{3}$, $x - \frac{\pi}{3} \neq k\pi$, $D(f) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{3} + k\pi, k \in \mathbb{Z}\right\}$

$0 < x - \frac{\pi}{3} < \pi \Rightarrow \frac{\pi}{3} < x < \frac{4\pi}{3}$; $p = \frac{4\pi}{3} - \frac{\pi}{3} = \pi$

$t = 2x$, $2x \neq \frac{\pi}{2} + k\pi$, $D(g) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

$-\frac{\pi}{2} < 2x < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{4} < x < \frac{3\pi}{4}$; $p = \frac{3\pi}{4} - \left(-\frac{\pi}{4}\right) = \pi$

$t = x + \frac{\pi}{4}$, $x + \frac{\pi}{4} \neq k\pi$, $D(h) = \left\{x \in \mathbb{R} \mid x \neq -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}\right\}$

$0 < x + \frac{\pi}{4} < 2\pi \Rightarrow -\frac{\pi}{4} < x < \frac{7\pi}{4}$; $p = \frac{7\pi}{4} - \left(-\frac{\pi}{4}\right) = 2\pi$

176. a) $2 - m \geqslant 0 \Rightarrow m \leqslant 2$

b) $3m - 2 \leqslant -1 \Rightarrow m \leqslant \frac{1}{3}$ ou $3m - 2 \geqslant 1 \Rightarrow m \geqslant 1$

c) $\frac{2m - 1}{1 - 3m} \leqslant -1 \Rightarrow \frac{-m}{1 - 3m} \leqslant 0 \Rightarrow 0 \leqslant m < \frac{1}{3}$

ou

$\frac{2m - 1}{1 - 3m} \geqslant 1 \Rightarrow \frac{5m - 2}{1 - 3m} \geqslant 0 \Rightarrow \frac{1}{3} < m \leqslant \frac{2}{5}$

177. $$\frac{1}{\left(1 + \frac{1}{\cos x}\right)} \cdot \frac{(1 + \cos x)^{\frac{1}{2}}}{(1 - \cos x)^{\frac{1}{2}}} = \frac{\cos x}{\sqrt{1 - \cos^2 x}} = \frac{\cos x}{\sqrt{\operatorname{sen}^2 x}} = \frac{\cos x}{|\operatorname{sen} x|}$$

178. $$\frac{\frac{1}{\operatorname{sen} x} - \operatorname{sen} x}{\frac{1}{\cos x} - \cos x} = \frac{\cos^2 x}{\operatorname{sen} x} \cdot \frac{\cos x}{\operatorname{sen}^2 x} = \operatorname{cotg}^3 x$$

179. $$\frac{1 - \operatorname{sen}^2 \theta}{1 - \operatorname{sen} \theta} = \frac{(1 + \operatorname{sen} \theta)(1 - \operatorname{sen} \theta)}{1 - \operatorname{sen} \theta} = 1 + \operatorname{sen} \theta$$

180. $$\frac{\overset{1}{\cancel{(\cos^2 x + \operatorname{sen}^2 x)}}(\cos^2 x - \operatorname{sen}^2 x)}{(1 + \operatorname{tg}^2 x)(1 - \operatorname{tg}^2 x)} = \frac{\cos^2 x - \cos^2 x \cdot \operatorname{tg}^2 x}{\sec^2 x \cdot (1 - \operatorname{tg}^2 x)} =$$

$$= \frac{\cos^2 x}{\sec^2 x} = \cos^4 x$$

181. $$\frac{\operatorname{sen}^2 x + (1 + \cos x)^2}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2 + 2 \cos x}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2}{\operatorname{sen} x} = 2 \operatorname{cossec} x$$

182. $\operatorname{sen} x - \operatorname{cossec} x = t \Rightarrow (\operatorname{sen} x - \operatorname{cossec} x)^2 = t^2 \Rightarrow$

$\Rightarrow \operatorname{sen}^2 x + \operatorname{cossec}^2 x = t^2 + 2$

183. $$\frac{\sec^2 x}{\operatorname{cossec}^2 x} = \frac{\operatorname{sen}^2 x}{1 - \operatorname{sen}^2 x} = \frac{\left(\frac{n-1}{n}\right)^2}{1 - \left(\frac{n-1}{n}\right)^2} = \frac{(n-1)^2}{2n - 1}$$

184. a) $\operatorname{tg}(-x) = \dfrac{\operatorname{sen}(-x)}{\cos(-x)} = -\dfrac{\operatorname{sen} x}{\cos x} = -\operatorname{tg}(x)$; a função é ímpar

b) $\operatorname{cotg}(-x) = \dfrac{1}{\operatorname{tg}(-x)} = -\dfrac{1}{\operatorname{tg} x} = -\operatorname{cotg}(x)$; a função é ímpar

c) $\sec(-x) = \dfrac{1}{\cos(-x)} = \dfrac{1}{\cos x} = \sec x$; a função é par

d) $\operatorname{cossec}(-x) = \dfrac{1}{\operatorname{sen}(-x)} = -\dfrac{1}{\operatorname{sen} x} = -\operatorname{cossec} x$; a função é ímpar

185. a) $0 \in D(f)$, f é ímpar $\Rightarrow f(-0) = -f(0) \Rightarrow f(0) + f(0) = 0 \Rightarrow f(0) = 0$

b) $$\left.\begin{array}{l} f \text{ é ímpar} \Rightarrow f(-x) = -f(x) \\ f \text{ é par} \Rightarrow f(-x) = f(x) \end{array}\right\} \Rightarrow -f(x) = f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0, \forall x$$

186. $f(x) = f(-x) = 3 \Rightarrow f(x)$ é par; $g(x)$ é par, $\forall n$, pois

$$g(x) = \underbrace{f(x) \cdot f(x) \cdot \ldots \cdot f(x)}_{n \text{ fatores}} = \underbrace{f(-x) \cdot f(-x) \cdot f(-x)}_{n \text{ fatores}} \cdot \ldots \cdot f(-x) = g(-x), \forall x$$

CAPÍTULO IX — Transformações

188. $\text{cotg}\,(120° + 45°) = \dfrac{-\dfrac{\sqrt{3}}{3} \cdot 1 - 1}{-\dfrac{\sqrt{3}}{3} + 1} = -(2 + \sqrt{3});$

$\cos\,(225° + 30°) = -\dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{2} - \sqrt{6}}{4};$

$\sec 225° = \dfrac{1}{\cos 255°} = -\sqrt{2} - \sqrt{6}$

$\text{sen}\,(45° - 30°) = \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6} - \sqrt{2}}{4};$

$\text{cossec}\,15° = \dfrac{1}{\text{sen}\,15°} = \sqrt{6} + \sqrt{2}$

189. $\text{tg}\,(A - B) = \dfrac{2 - 1}{1 + 2 \cdot 1} = \dfrac{1}{3}$

190. $\left.\begin{aligned} \text{sen}\,(60° + 45°) &= \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{6}}{4} + \dfrac{\sqrt{2}}{4}\ (A) \\ \cos\,(30° + 45°) &= \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4}\ (B) \end{aligned}\right\}\ A - B = \dfrac{\sqrt{2}}{2}$

193. $\cos x = +\sqrt{1 - \text{sen}^2\,x} = \dfrac{8}{17};\ \cos y = -\sqrt{1 - \text{sen}^2\,y} = -\dfrac{4}{5};$

$\text{tg}\,x = \dfrac{15}{8};\ \text{tg}\,y = \dfrac{3}{4}$

$\text{sen}\,(x + y) = \dfrac{15}{17} \cdot \left(-\dfrac{4}{5}\right) + \left(-\dfrac{3}{5}\right)\left(\dfrac{8}{17}\right) = -\dfrac{84}{85};$

$\cos\,(x + y) = \dfrac{13}{85};\ \text{tg}\,(x + y) = -\dfrac{84}{13}$

195. a) $f(x) = \cos 2x \cdot \cos 2x - \text{sen}\,2x \cdot \text{sen}\,2x = \cos 4x$

$D(f) = \mathbb{R};\ Im(f) = [-1, 1];\ p = \dfrac{\pi}{2}$

b) $g(x) = 2 \cdot \text{sen}\,\dfrac{\pi}{3} \cos x - 2 \cos \dfrac{\pi}{3}\,\text{sen}\,x = 2 \cdot \text{sen}\left(\dfrac{\pi}{3} - x\right)$

$D(g) = \mathbb{R};\ Im(g) = [-2, 2];\ p = 2\pi$

c) $h(x) = \dfrac{\dfrac{\text{sen}\,x}{\cos x} + \dfrac{\cos x}{\cos x}}{\dfrac{\cos x}{\cos x} - \dfrac{\text{sen}\,x}{\cos x}} = \dfrac{\text{tg}\,x + 1}{1 - \text{tg}\,x} = \text{tg}\left(x + \dfrac{\pi}{4}\right)$

$$x + \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi \Rightarrow D(h) = \left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\pi\right\}$$

$$\left.\begin{array}{l} x + \frac{\pi}{4} = -\frac{\pi}{2} \Rightarrow x = -\frac{3\pi}{4} \\ x + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4} \end{array}\right\} \Rightarrow p = \frac{\pi}{4} - \left(-\frac{3\pi}{4}\right) = \pi$$

196. $f(x) = \text{sen } x\,(\cos 2x \cos 3x - \text{sen } 2x \text{ sen } 3x) +$
$+ \cos x\,(\text{sen } 2x \cos 3x + \text{sen } 3x \cos 2x) \Rightarrow$
$\Rightarrow f(x) = \text{sen } x \cos 5x + \cos x \text{ sen } 5x = \text{sen } 6x;\ p = \frac{2\pi}{6} = \frac{\pi}{3}$

197. $\text{tg}\,(75° - 60°) = \dfrac{(2 + \sqrt{3}) - \sqrt{3}}{1 + (2 + \sqrt{3})\sqrt{3}} = 2 - \sqrt{3}$

199. $\text{tg } x + \text{cotg } x = 3 \Rightarrow \dfrac{\text{sen}^2 x + \cos^2 x}{\cos x \cdot \text{sen } x} = 3 \Rightarrow \text{sen } x \cdot \cos x = \dfrac{1}{3} \Rightarrow$
$\Rightarrow 2 \cdot \text{sen } x \cdot \cos x = \dfrac{2}{3} \Rightarrow \text{sen } 2x = \dfrac{2}{3}$

201. a) $\text{sen}\left(\dfrac{\pi}{2} + 2\alpha\right) = \cos 2\alpha = 1 - 2\,\text{sen}^2\,\alpha \Rightarrow \text{sen}\left(\dfrac{\pi}{2} + 2\alpha\right) = \dfrac{1}{9}$

b) $$\left.\begin{array}{l} \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\,\text{sen}\,\alpha \\ \cos\alpha = \sqrt{1 - \text{sen}^2\,\alpha} \Rightarrow \cos\alpha = \frac{\sqrt{5}}{3} \end{array}\right\} \Rightarrow \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{10} - 2\sqrt{2}}{6}$$

203. $\text{sen } x = -\sqrt{1 - \left(\dfrac{3}{5}\right)^2} \Rightarrow \text{sen } x = -\dfrac{4}{5};$

$\text{sen } 3x = 3 \cdot \left(-\dfrac{4}{5}\right) - 4 \cdot \left(-\dfrac{4}{5}\right)^3 = -\dfrac{44}{125}$

204. $\cos x = -\sqrt{1 - \left(\dfrac{12}{13}\right)^2} \Rightarrow \cos x = -\dfrac{5}{13};$

$\cos 3x = 4 \cdot \left(-\dfrac{5}{13}\right)^3 - 3 \cdot \left(-\dfrac{5}{13}\right) = \dfrac{2\,035}{2\,197}$

205. $\text{tg } x = \sqrt{\sec^2 x - 1} = \dfrac{\sqrt{7}}{3} \Rightarrow \text{tg } 3x = \dfrac{3 \cdot \frac{\sqrt{7}}{3} - \left(\frac{\sqrt{7}}{3}\right)^3}{1 - 3\left(\frac{\sqrt{7}}{3}\right)^2} = -\dfrac{5\sqrt{7}}{9}$

206. $\left(\text{sen}^2\dfrac{\pi}{12} - \cos^2\dfrac{\pi}{12}\right) + \text{tg}\dfrac{\pi}{3} + \text{tg}\dfrac{14\pi}{3} = -\cos\left(2 \cdot \dfrac{\pi}{12}\right) + \text{tg}\dfrac{\pi}{3} + \text{tg}\dfrac{2\pi}{3} =$
$= -\cos\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{2}$

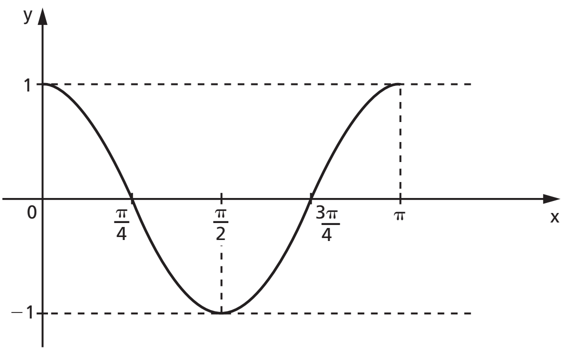
208. a) $f(x) = (\cos^2 x + \operatorname{sen}^2 x)(\cos^2 x - \operatorname{sen}^2 x) = \cos 2x$

x	t = 2x	cos t
0	0	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1

$\mathrm{Im}(f) = [-1, 1]$

$D(f) = \mathbb{R}$

$p = \pi$



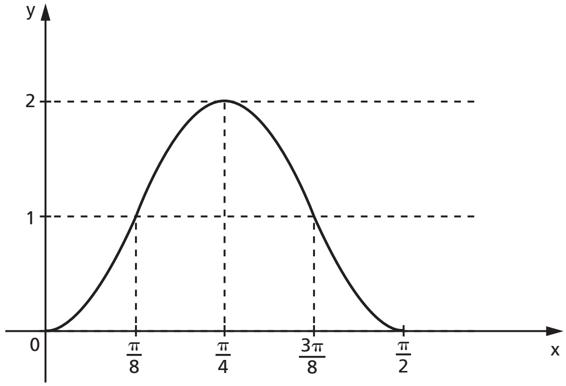
b) $g(x) = 2(2 \operatorname{sen} x \cos x)^2 = 2 \operatorname{sen}^2 2x = 1 - \cos 4x$

x	t = 4x	cos t	y = 1 − cos 4x
0	0	1	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{4}$	π	-1	2
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	1
$\frac{\pi}{2}$	2π	1	0

$\mathrm{Im}(g) = [0, 2]$

$D(g) = \mathbb{R}$

$p = \frac{\pi}{2}$



c) $h(x) = (\cos^2 x + \text{sen}^2 x)^2 - 2 \cdot \cos^2 x \cdot \text{sen}^2 x = 1 - 2 \cdot \left(\dfrac{\text{sen } 2x}{2}\right)^2 =$

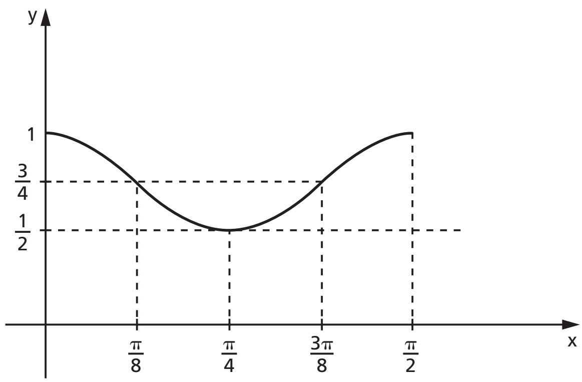
$= 1 - \dfrac{1}{2} \cdot \text{sen}^2 2x = 1 - \dfrac{1}{2} \cdot \left(\dfrac{1 - \cos 4x}{2}\right) = \dfrac{3}{4} + \dfrac{1}{4} \cdot \cos 4x$

x	t = 4x	cos t	y = h(x)
0	0	1	1
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{4}$	π	−1	$\frac{1}{2}$
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{2}$	2π	1	1

$\text{Im}(h) = \left[\dfrac{1}{2}, 1\right]$

$D(h) = \mathbb{R}$

$p = \dfrac{\pi}{2}$



209. a) $f(x) = \frac{1}{2}\text{sen } 2x; \quad p = \frac{2\pi}{2} \Rightarrow p = \pi$

b) $g(x) = \frac{1 - \text{tg}^2\, 2x}{\sec^2 2x} = (1 - \text{tg}^2\, 2x) \cdot \cos^2 2x = \cos^2 2x - \text{sen}^2\, 2x = \cos 4x$

$p = \frac{2\pi}{4} \Rightarrow p = \frac{\pi}{2}$

c) $h(x) = (\cos^2 x + \text{sen}^2\, x)(\cos^4 x - \cos^2 x \cdot \text{sen}^2\, x + \text{sen}^4\, x) =$
$= (\cos^4 x + \text{sen}^4\, x) - \cos^2 x \cdot \text{sen}^2\, x =$
$= (\cos^2 x + \text{sen}^2\, x)^2 - 3 \cdot \cos^2 x \cdot \text{sen}^2\, x =$
$= 1 - \frac{3}{4}(\text{sen } 2x)^2 = 1 - \frac{3}{4}\left(\frac{1 - \cos 4x}{2}\right) = \frac{5}{8} + \frac{3}{8} \cdot \cos 4x$

$p = \frac{2\pi}{4} = \frac{\pi}{2}$

210. $\text{sen } 2a = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}; \cos 2a = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25};$

$\text{sen } 2a + \cos 2a = \frac{31}{25}$

211. $\sec a = \frac{1}{\cos a} \Rightarrow \cos a = -\frac{2}{3}; \text{sen } a = \sqrt{1 - \left(-\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{3};$

$\text{sen } b = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}}; \text{sen } 2a = 2 \cdot \frac{\sqrt{5}}{3} \cdot \left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9};$

$\cos 2b = \left(\frac{1}{3}\right)^2 - \left(\sqrt{\frac{8}{9}}\right)^2 = \frac{1}{9} - \frac{8}{9} = -\frac{7}{9} \Rightarrow \cos 2b = -\frac{7}{9}$

212. $\text{tg } x = a \text{ cotg } x + b \cdot \frac{1}{\text{tg } 2x} \Rightarrow \text{tg } x = \frac{a}{\text{tg } x} + \frac{b(1 - \text{tg}^2\, x)}{2 \text{ tg } x} \Rightarrow$

$\Rightarrow 2\,\text{tg}^2\, x - 2a = -b\,\text{tg}^2\, x + b \Rightarrow (2 = -b \text{ e } -2a = b) \Rightarrow b = -2 \text{ e } a = 1$

215. $\cos\theta = -\sqrt{1 - \text{sen}^2\,\theta} \Rightarrow \cos\theta = -\frac{4}{5}; \text{sen } \frac{\theta}{2} = +\sqrt{\frac{1 - \cos\theta}{2}} \Rightarrow$

$\Rightarrow \text{sen } \frac{\theta}{2} = \frac{3}{\sqrt{10}}; A = 25 \cdot \frac{3}{5} + \sqrt{10} \cdot \frac{3}{\sqrt{10}} \Rightarrow A = 18$

216. $\cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \cos a_n}{2}} \Rightarrow \cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \frac{n}{n+1}}{2}} \Rightarrow$

$\Rightarrow \cos\left(\frac{a_n}{2}\right) = \sqrt{\frac{2n+1}{2n+2}} \Rightarrow$

$\Rightarrow \cos\left(\frac{a_n}{2}\right) = \frac{\sqrt{4n^2 + 6n + 2}}{2n + 2}$

218. $\text{sen}\,\dfrac{x}{2} = +\sqrt{\dfrac{1-\cos x}{2}} = \dfrac{\sqrt{3}}{3}$; $\text{tg}\,\dfrac{x}{2} = +\sqrt{\dfrac{1-\cos x}{1+\cos x}} = \dfrac{\sqrt{2}}{2}$

219. $\cos\dfrac{x}{2} = +\sqrt{\dfrac{1+\cos x}{2}} = \dfrac{7}{5\sqrt{2}}$; $\text{sen}\,\dfrac{x}{4} = +\sqrt{\dfrac{1-\cos\dfrac{x}{2}}{2}} =$

$= +\sqrt{\dfrac{10-7\sqrt{2}}{20}}$

$\cos\dfrac{x}{4} = +\sqrt{\dfrac{1+\cos\dfrac{x}{2}}{2}} = \sqrt{\dfrac{10+7\sqrt{2}}{20}}$; $\text{tg}\,\dfrac{x}{4} = \sqrt{\dfrac{10-7\sqrt{2}}{10+7\sqrt{2}}} =$

$= 5\sqrt{2} - 7$

220. $\sec x = 4 \Rightarrow \cos x = \dfrac{1}{4}$

$\dfrac{3\pi}{2} < x < 2\pi \Rightarrow \dfrac{3\pi}{4} < \dfrac{x}{2} < \pi$

$\text{tg}\left(\dfrac{\pi}{2} + \dfrac{x}{2}\right) = -\text{cotg}\,\dfrac{x}{2} = -\left(-\sqrt{\dfrac{1+\cos x}{1-\cos x}}\right) = \dfrac{\sqrt{15}}{3}$

222. $f(x) = \dfrac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}} = \sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}} = |\text{tg}\,x|$

$\text{Im}(f) = \mathbb{R}_+$; $p = \pi$; $D(f) = \left\{x \in \mathbb{R} \mid x \neq \dfrac{\pi}{2} + k\pi\right\}$

223. $f(x) = \sqrt{1+\cos 4x} = \sqrt{2}\cdot\sqrt{\dfrac{1+\cos 4x}{2}} = \sqrt{2}\cdot|\cos 2x|$

$p = \dfrac{\pi}{2}$

224. $\text{tg}\,a = \dfrac{2\cdot\text{tg}\,\dfrac{a}{2}}{1-\text{tg}^2\,\dfrac{a}{2}} = \dfrac{2\cdot\dfrac{1}{2}}{1-\left(\dfrac{1}{2}\right)^2} = \dfrac{4}{3}$

225. $\text{cotg}\,\dfrac{a}{2} = \dfrac{1}{\text{tg}\,\dfrac{a}{2}} \Rightarrow \text{tg}\,\dfrac{a}{2} = \dfrac{\sqrt{3}}{3}$; $\text{sen}\,a = \dfrac{2\cdot\text{tg}\,\dfrac{a}{2}}{1+\text{tg}^2\,\dfrac{a}{2}} = \dfrac{\sqrt{3}}{2}$

229. a) $y = 2\,\text{sen}\left(\dfrac{a+b+c-a+b-c}{2}\right)\cdot\cos\left(\dfrac{a+b+c+a-b+c}{2}\right) =$

$= 2\,\text{sen}\,b\cos(a+c)$

b) $y = 2\cdot\cos\left(\dfrac{a+2b+a}{2}\right)\cdot\cos\left(\dfrac{a+2b-a}{2}\right) = 2\cos(a+b)\cos b$

c) $y = [\text{sen}\,(a + 3r) + \text{sen}\,a] + [\text{sen}\,(a + 2r) + \text{sen}\,(a + r)] =$
$= 2 \cdot \text{sen}\,\frac{2a + 3r}{2} \cdot \cos\frac{3r}{2} + 2 \cdot \text{sen}\,\frac{2a + 3r}{2} \cdot \cos\frac{r}{2} =$
$= 2 \cdot \text{sen}\,\frac{2a + 3r}{2} \cdot \left(\cos\frac{3r}{2} + \cos\frac{r}{2}\right) = 4 \cdot \text{sen}\,\frac{2a + 3r}{2} \cdot \cos r \cdot \cos\frac{r}{2}$

d) $y = [\cos\,(a + 3b) + \cos a] + [\cos\,(a + 2b) + \cos\,(a + b)] =$
$= 2 \cdot \cos\frac{2a + 3b}{2} \cdot \cos\frac{3b}{2} + 2 \cdot \cos\frac{2a + 3b}{2} \cdot \cos\frac{b}{2} =$
$= 2 \cdot \cos\frac{2a + 3b}{2} \cdot \left(\cos\frac{3b}{2} + \cos\frac{b}{2}\right) = 4 \cdot \cos\frac{2a + 3b}{2} \cdot \cos b \cdot \cos\frac{b}{2}$

e) $y = (\cos p + \cos q)(\cos p - \cos q)$
$y = 2 \cdot \cos\left(\frac{p + q}{2}\right) \cdot \cos\left(\frac{p - q}{2}\right)\left[-2 \cdot \text{sen}\left(\frac{p + q}{2}\right) \cdot \text{sen}\left(\frac{p - q}{2}\right)\right] =$
$= -\text{sen}\,(p + q) \cdot \text{sen}\,(p - q)$

f) $y = (\text{sen}\,p + \text{sen}\,q)(\text{sen}\,p - \text{sen}\,q)$
$y = 2 \cdot \text{sen}\left(\frac{p + q}{2}\right) \cdot \cos\left(\frac{p - q}{2}\right) \cdot 2\,\text{sen}\left(\frac{p - q}{2}\right) \cdot \cos\left(\frac{p + q}{2}\right) =$
$= \text{sen}\,(p + q) \cdot \text{sen}\,(p - q)$

g) $y = \frac{1 + \cos 2p}{2} - \frac{1 - \cos 2q}{2} = \frac{1}{2}(\cos 2p + \cos 2q) =$
$= \cos\,(p + q) \cdot \cos\,(p - q)$

h) $y = \frac{2\,\text{sen}\,(a + b)\cos\,(a - b)}{-2\,\text{sen}\,(a + b)\,\text{sen}\,(a - b)} = -\text{cotg}\,(a - b)$

i) $y = \dfrac{\text{sen}\,\frac{\pi}{2} + \text{sen}\,a}{\text{sen}\,\frac{\pi}{2} - \text{sen}\,a} = \dfrac{2\,\text{sen}\left(\frac{\pi}{4} + \frac{a}{2}\right) \cdot \cos\left(\frac{\pi}{4} - \frac{a}{2}\right)}{2\,\text{sen}\left(\frac{\pi}{4} - \frac{a}{2}\right) \cdot \cos\left(\frac{\pi}{4} + \frac{a}{2}\right)} =$
$= \text{tg}\left(\frac{\pi}{4} + \frac{a}{2}\right) \cdot \text{cotg}\left(\frac{\pi}{4} - \frac{a}{2}\right)$

231. a) $\frac{p + q}{2} = \frac{7\pi}{8}$; $\frac{p - q}{2} = \frac{\pi}{8}$; $p = \pi$ e $q = \frac{3\pi}{4}$;
$y = \frac{1}{2} \cdot \left(2\cos\frac{7\pi}{8} \cdot \cos\frac{\pi}{8}\right) = \frac{1}{2}\left(\cos\pi + \cos\frac{3\pi}{4}\right) = \frac{-2 - \sqrt{2}}{4}$

b) $\frac{p + q}{2} = \frac{13\pi}{12}$; $\frac{p - q}{2} = \frac{7\pi}{12}$; $p = \frac{5\pi}{3}$ e $q = \frac{\pi}{2}$;
$y = -\frac{1}{2}\left(-2\,\text{sen}\,\frac{13\pi}{12} \cdot \text{sen}\,\frac{7\pi}{12}\right) = -\frac{1}{2}\left(\cos\frac{5\pi}{3} - \cos\frac{\pi}{2}\right) = -\frac{1}{4}$

c) $\frac{p + q}{2} = \frac{5\pi}{24}$; $\frac{p - q}{2} = \frac{\pi}{24}$; $p = \frac{\pi}{4}$ e $q = \frac{\pi}{6}$;
$y = \frac{1}{2}\left(2\,\text{sen}\,\frac{5\pi}{24} \cdot \cos\frac{\pi}{24}\right) = \frac{1}{2}\left(\text{sen}\,\frac{\pi}{4} + \text{sen}\,\frac{\pi}{6}\right) = \frac{1 + \sqrt{2}}{4}$

233. $(\text{tg } 81° + \text{tg } 9°) - (\text{tg } 63° + \text{tg } 27°) =$

$$= \frac{\text{sen } 90°}{\cos 81° \cdot \cos 9°} - \frac{\text{sen } 90°}{\cos 63° \cdot \cos 27°} =$$

$$= \frac{1}{\frac{1}{2}(\cos 90° + \cos 72°)} - \frac{1}{\frac{1}{2}(\cos 90° + \cos 36°)} =$$

$$= \frac{2}{\text{sen } 18°} - \frac{2}{\text{sen } 54°} = \frac{2\,(\text{sen } 54° - \text{sen } 18°)}{\text{sen } 18° \cdot \text{sen } 54°} =$$

$$= \frac{2 \cdot 2 \cdot \text{sen } 18° \cdot \cos 36°}{\text{sen } 18° \cdot \text{sen } 54°} = 4$$

235. $f(x) = \text{sen } 2x + \text{sen}\left(\frac{\pi}{2} + 2x\right) = 2\,\text{sen}\left(2x + \frac{\pi}{4}\right) \cdot \cos\left(-\frac{\pi}{4}\right)$

$f(x) = \sqrt{2} \cdot \text{sen}\left(2x + \frac{\pi}{4}\right)$; $D(f) = \mathbb{R}$; $Im(f) = [-\sqrt{2}, \sqrt{2}]$, $p = \pi$

236. $$f(x) = \frac{\text{tg}\frac{\pi}{4} + \text{tg } x}{\text{tg}\frac{\pi}{4} - \text{tg } x} = \frac{\dfrac{\text{sen}\left(\frac{\pi}{4} + x\right)}{\cos\frac{\pi}{4} \cdot \cos x}}{\dfrac{\text{sen}\left(\frac{\pi}{4} - x\right)}{\cos\frac{\pi}{4} \cdot \cos x}} = \frac{\text{sen}\left(\frac{\pi}{4} + x\right)}{\text{sen}\left(\frac{\pi}{4} - x\right)} = \text{tg}\left(\frac{\pi}{4} + x\right);\ p = \pi$$

237. $|\text{sen } x - \text{sen } y| = \left|2\,\text{sen}\,\frac{x-y}{2} \cdot \cos\frac{x+y}{2}\right| =$

$$= |2| \cdot \left|\text{sen}\,\frac{x-y}{2}\right| \cdot \left|\cos\frac{x+y}{2}\right| \leqslant |2| \cdot \left|\frac{x-y}{2}\right| \cdot 1 =$$

$$= \left|2 \cdot \left(\frac{x-y}{2}\right)\right| = |x - y| \Rightarrow |\text{sen } x - \text{sen } y| \leqslant |x - y|$$

238. a) $\text{tg}\,b\,(\text{tg}\,a + \text{tg}\,c) + \text{tg}\,c \cdot \text{tg}\,a = \text{cotg}\,(a + c) \cdot (\text{tg}\,a + \text{tg}\,c) + \text{tg}\,c \cdot \text{tg}\,a =$

$$= \frac{\cos(a+c)}{\text{sen}(a+c)} \cdot \frac{\text{sen}(a+c)}{\cos a \cos c} + \frac{\text{sen } c \text{ sen } a}{\cos c \cos a} =$$

$$= \frac{\cos a \cos c - \text{sen } a \text{ sen } c + \text{sen } c \text{ sen } a}{\cos a \cos c} = 1$$

b) $\cos^2 a + \cos^2 b + \cos^2 c - 2 \cdot \text{sen } a \cdot \text{sen } b \cdot \text{sen } c =$

$$= \frac{1 + \cos 2a}{2} + \frac{1 + \cos 2b}{2} + (1 - \text{sen}^2 c) + (-2 \cdot \text{sen}\,a \cdot \text{sen}\,b) \cdot \text{sen}\,c =$$

$= 1 + \cos(a + b)\cos(a - b) + 1 - \text{sen}^2 c + [\cos(a + b) -$
$- \cos(a - b)]\,\text{sen } c =$
$= 2 + \text{sen}\,c \cdot \cos(a - b) - \text{sen}^2 c + [\text{sen}\,c - \cos(a - b)]\,\text{sen}\,c = 2$

241. $\text{sen } 4A + \text{sen } 4B = -\text{sen } 4C \Rightarrow$
$\Rightarrow 2 \text{ sen } (2A + 2B) \cdot \cos (2A - 2B) = -2 \text{ sen } 2C \cdot \cos 2C \Rightarrow$
$\Rightarrow \text{sen } (360° - 2C) \cdot \cos (2A - 2B) = -\text{sen } 2C \cdot \cos 2C \Rightarrow$
$\Rightarrow \cos (2A - 2B) = \cos 2C \Rightarrow 2A - 2B = 2C \Rightarrow$
$\Rightarrow \left(A = B + C = \frac{\pi}{2} \text{ ou } 2A - 2B = -2C\right) \Rightarrow B = A + C = \frac{\pi}{2}$

242. $A + B + C = \pi \Rightarrow C = \pi - (A + B) \Rightarrow 3C = 3\pi - (3A + 3B) \Rightarrow$
$\Rightarrow \text{sen } 3C = -\text{sen } (3A + 3B)$, então:
$(\text{sen } 3A + \text{sen } 3B) + \text{sen } 3C = 0 \Rightarrow$

$$\Rightarrow 2 \cdot \text{sen}\frac{3(A+B)}{2} \cdot \cos\frac{3(A-B)}{2} - 2 \cdot \text{sen}\frac{3(A+B)}{2} \cdot \cos\frac{3(A+B)}{2} = 0 \Rightarrow$$

$$\Rightarrow 2 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \left[\cos \frac{3(A-B)}{2} - \cos \frac{3(A+B)}{2}\right] = 0 \Rightarrow$$

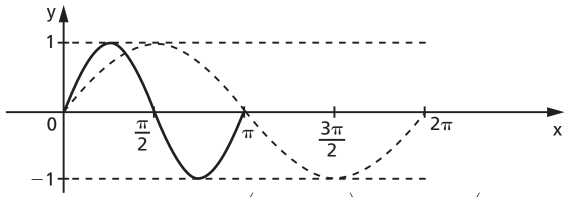
$$\Rightarrow 4 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \text{sen } \frac{3A}{2} \cdot \text{sen } \frac{3B}{2} = 0 \Rightarrow$$

$$\Rightarrow \text{sen } \frac{3(A+B)}{2} = 0 \text{ ou sen } \frac{3A}{2} = 0 \text{ ou sen } \frac{3B}{2} = 0 \Rightarrow$$

$$\Rightarrow C = \frac{\pi}{3} \text{ ou } A = \frac{\pi}{3} \text{ ou } B = \frac{\pi}{3} \text{ (respectivamente)}$$

244. $4 \text{ sen } (x + 60°) \cdot \cos (x + 30°) =$
$= 4\,(\text{sen } x \cos 60° + \text{sen } 60° \cos x) \cdot (\cos x \cos 30° - \text{sen } x \text{ sen } 30°) =$
$= \left(\text{sen } x + \sqrt{3} \cos x\right) \cdot \left(\sqrt{3} \cos x - \text{sen } x\right) = \left(\sqrt{3} \cos x\right)^2 - (\text{sen } x)^2 =$
$= 3 \cos^2 x - \text{sen}^2 x$

245. a) $f(x) = \text{sen } 2x$, $Im(f) = [-1, 1]$, $D(f) = \mathbb{R}$, $p = \pi$



b) $f(x) = \text{sen } 2x + \text{sen } \left(2x + \frac{\pi}{2}\right) = 2 \text{ sen } \left(2x + \frac{\pi}{4}\right) \cdot \cos \left(-\frac{\pi}{4}\right) =$
$= \sqrt{2} \text{ sen } \left(2x + \frac{\pi}{4}\right)$

246. $\text{sen } u + \cos u = \text{sen } u + \text{sen}\left(\frac{\pi}{2} - u\right) = \sqrt{2} \cdot \cos\left(u - \frac{\pi}{4}\right) = \sqrt{2} \cos v \,(1)$

$$\sqrt{2} \cdot \text{sen } u \cdot \cos u = \frac{2 \cdot \text{sen } u \cdot \cos u}{\sqrt{2}} = \frac{\text{sen } 2u}{\sqrt{2}} = \frac{\text{sen}\left(\frac{\pi}{2} - 2v\right)}{\sqrt{2}} = \frac{\cos 2v}{\sqrt{2}}$$

$$S = \frac{\sqrt{2} \cdot \cos v}{\frac{\cos 2v}{\sqrt{2}}} = \frac{2 \cdot \cos v}{\cos 2v} = \frac{2x}{2x^2 - 1}$$

247. $n < 20 \cdot \cos^2 15 = 20 \cdot \dfrac{1 + \cos 30°}{2} = 20 \cdot \left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{4}\right) \cong 18{,}66;$
então $n = 18$

248. $f(x) = \cos 2x + \operatorname{sen} 2x = \operatorname{sen}\left(\dfrac{\pi}{2} + 2x\right) + \operatorname{sen} 2x = \sqrt{2}\operatorname{sen}\left(\dfrac{\pi}{4} + 2x\right)$
$\operatorname{Im}(f) = [-\sqrt{2}, \sqrt{2}]$

CAPÍTULO X — Identidades

253. a) $f(x) = (\cos^2 x + \operatorname{sen}^2 x)^2 = 1 = g(x)$

b) $f(x) = \dfrac{\operatorname{sen} x}{\operatorname{cossec} x} + \dfrac{\cos x}{\sec x} = \dfrac{\operatorname{sen} x}{\dfrac{1}{\operatorname{sen} x}} + \dfrac{\cos x}{\dfrac{1}{\cos x}} =$
$= \operatorname{sen}^2 x + \cos^2 x = 1 = g(x)$

254. $f(x) = \operatorname{tg} x + \operatorname{cotg} x = \dfrac{\operatorname{sen} x}{\cos x} + \dfrac{\cos x}{\operatorname{sen} x} = \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x} =$
$= \dfrac{1}{\cos x} \cdot \dfrac{1}{\operatorname{sen} x} = \sec x \cdot \operatorname{cossec} x = g(x)$

255. $f(x) = \left(\dfrac{\operatorname{sen} x}{\cos x} + \dfrac{\cos x}{\operatorname{sen} x}\right)\left(\dfrac{1}{\cos x} - \cos x\right)\left(\dfrac{1}{\operatorname{sen} x} - \operatorname{sen} x\right) =$
$= \left(\dfrac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen} x \cos x}\right)\left(\dfrac{1 - \cos^2 x}{\cos x}\right)\left(\dfrac{1 - \operatorname{sen}^2 x}{\operatorname{sen} x}\right) =$
$= \dfrac{\operatorname{sen}^2 x \cdot \cos^2 x}{\operatorname{sen}^2 x \cdot \cos^2 x} = 1 = g(x)$

256. $f(x) = \dfrac{1}{\cos^2 x} + \dfrac{1}{\operatorname{sen}^2 x} = \dfrac{\operatorname{sen}^2 x + \cos^2 x}{\cos^2 x \cdot \operatorname{sen}^2 x} = \dfrac{1}{\cos^2 x} \cdot \dfrac{1}{\operatorname{sen}^2 x} =$
$= \sec^2 x \cdot \operatorname{cossec}^2 x = g(x)$

257. $f(x) = \dfrac{\operatorname{cotg}^2 x}{\operatorname{cossec}^2 x} = \dfrac{\cos^2 x}{\operatorname{sen}^2 x} : \dfrac{1}{\operatorname{sen}^2 x} = \cos^2 x = g(x)$

258. $f(x) = \dfrac{(\operatorname{sen} x - \cos x)(\operatorname{sen}^2 x + \cos^2 x + \operatorname{sen} x \cos x)}{\operatorname{sen} x - \cos x} = 1 + \operatorname{sen} x \cos x = g(x)$

259. $f(x) = 1 + \operatorname{cotg}^2 x + \operatorname{tg}^2 x = \sec^2 x + \operatorname{cotg}^2 x = g(x)$

260. $f(x) = 2\left(\operatorname{sen} x + \dfrac{\operatorname{sen} x}{\cos x}\right)\left(\cos x + \dfrac{\cos x}{\operatorname{sen} x}\right) =$
$= 2\left[\dfrac{\operatorname{sen} x\,(\cos x + 1)}{\cos x}\right]\left[\dfrac{\cos x\,(\operatorname{sen} x + 1)}{\operatorname{sen} x}\right] =$
$= 2(\cos x + 1)(\operatorname{sen} x + 1) = h(x)$

$$g(x) = (1 + \text{sen } x + \cos x)^2 =$$
$$= 1 + \text{sen}^2 x + \cos^2 x + 2 \text{ sen } x \cos x + 2 \text{ sen } x + 2 \cos x =$$
$$= 2(1 + \text{sen } x + \cos x + \text{sen } x \cos x) = 2(1 + \text{sen } x)(1 + \cos x) = h(x)$$

261. $f(x) = 1 + 2 \text{ cotg } x + \text{cotg}^2 x + 1 - 2 \text{ cotg } x + \text{cotg}^2 x = 2 + 2 \text{ cotg}^2 x =$
$= 2(1 + \text{cotg}^2 x) = 2 \text{ cossec}^2 x = g(x)$

262. $f(x) = \dfrac{(1 - \cos^2 x)^2}{(1 - \text{sen}^2 x)^2} = \left(\dfrac{\text{sen}^2 x}{\cos^2 x}\right)^2 = \text{tg}^4 x = g(x)$

263. $f(x) - g(x) = \text{cotg}^2 x - 2 \text{ cotg } x \cdot \cos x + \cos^2 x + 1 - 2 \cdot \text{sen } x + \text{sen}^2 x -$
$- 1 + 2 \cdot \text{cossec } x - \text{cossec}^2 x =$
$$= -2 \cdot \frac{\cos x}{\text{sen } x} \cdot \cos x - 2 \cdot \text{sen } x + 2 \cdot \frac{1}{\text{sen } x} =$$
$$= \frac{-2 \cdot \cos^2 x - 2 \cdot \text{sen}^2 x + 2}{\text{sen } x} = 0$$

264. $f(x) - g(x) = \dfrac{\cos x + \cos y}{\text{sen } x - \text{sen } y} - \dfrac{\text{sen } x + \text{sen } y}{\cos y - \cos x} =$
$$= \frac{\cos^2 y - \cos^2 x - \text{sen}^2 x + \text{sen}^2 y}{(\text{sen } x - \text{sen } y)(\cos y - \cos x)} = 0$$

265. $f(x) = \dfrac{\cos x + \dfrac{\cos x}{\text{sen } x}}{\dfrac{\text{sen } x}{\cos x} + \dfrac{1}{\cos x}} = \dfrac{(\text{sen } x \cdot \cos x + \cos x) \cdot \cos x}{\text{sen } x \,(\text{sen } x + 1)} =$
$$= \frac{\cos^2 x \,(\text{sen } x + 1)}{\text{sen } x \,(\text{sen } x + 1)} = \cos x \cdot \text{cotg } x = g(x)$$

266. $f(x) = \dfrac{\text{sen}^2 x - \cos^2 y + \cos^2 x \cos^2 y}{\cos^2 x \cdot \cos^2 y} = \dfrac{\text{sen}^2 x - \cos^2 y \,(1 - \cos^2 x)}{\cos^2 x \cdot \cos^2 y} =$
$$= \frac{\text{sen}^2 x \,(1 - \cos^2 y)}{\cos^2 x \cdot \cos^2 y} = \text{tg}^2 x \cdot \text{tg}^2 y = g(x)$$

267. $g(x) = \text{cossec}^2 x - 2 \cdot \text{cossec } x \cdot \text{cotg } x + \text{cotg}^2 x =$
$$= \frac{1}{\text{sen}^2 x} - \frac{2 \cos x}{\text{sen}^2 x} + \frac{\cos^2 x}{\text{sen}^2 x} =$$
$$= \frac{(1 - \cos x)^2}{1 - \cos^2 x} = \frac{(1 - \cos x)^2}{(1 + \cos x)(1 - \cos x)} = f(x)$$

268. $f(x) = \dfrac{\dfrac{\cos x}{\text{sen } x} + \dfrac{\cos y}{\text{sen } y}}{\dfrac{\text{sen } x}{\cos x} + \dfrac{\text{sen } y}{\cos y}} =$
$$= \frac{(\cos x \cdot \text{sen } y + \cos y \cdot \text{sen } x)}{\text{sen } x \cdot \text{sen } y} \cdot \frac{(\cos x \cdot \cos y)}{(\text{sen } x \cdot \cos y + \text{sen } y \cdot \cos x)} =$$
$$= \text{cotg } x \cdot \text{cotg } y = g(x)$$

269. $f(x) = \sec^2 x \cdot \sec^2 y + 2 \cdot \sec x \cdot \sec y \cdot \mathrm{tg}\, x \cdot \mathrm{tg}\, y + \mathrm{tg}^2 x \cdot \mathrm{tg}^2 y =$
$= (1 + \mathrm{tg}^2 x)(1 + \mathrm{tg}^2 y) + 2 \cdot \sec x \cdot \sec y \cdot \mathrm{tg}\, x \cdot \mathrm{tg}\, y + \mathrm{tg}^2 x \cdot \mathrm{tg}^2 y =$
$= 1 + \mathrm{tg}^2 x + \mathrm{tg}^2 y + \mathrm{tg}^2 x \cdot \mathrm{tg}^2 y + 2 \cdot \sec x \cdot \sec y \cdot \mathrm{tg}\, x \cdot \mathrm{tg}\, y + \mathrm{tg}^2 x \cdot \mathrm{tg}^2 y =$
$= 1 + \mathrm{tg}^2 y\,(1 + \mathrm{tg}^2 x) + 2 \cdot \sec x \cdot \sec y \cdot \mathrm{tg}\, x \cdot \mathrm{tg}\, y + \mathrm{tg}^2 x\,(1 + \mathrm{tg}^2 y) =$
$= 1 + \mathrm{tg}^2 y \cdot \sec^2 x + 2 \cdot \sec x \cdot \sec y \cdot \mathrm{tg}\, x \cdot \mathrm{tg}\, y + \mathrm{tg}^2 x \cdot \sec^2 y = g(x)$

270. $f(x) - g(x) = \dfrac{(\sec x - \mathrm{tg}\, x)(\sec x + \mathrm{tg}\, x) - 1}{\sec x + \mathrm{tg}\, x} = \dfrac{\sec^2 x - \mathrm{tg}^2 x - 1}{\sec x + \mathrm{tg}\, x} = 0$

271. $f(x) = (\mathrm{cossec}^2 x - \mathrm{cotg}^2 x)(\mathrm{cossec}^4 x + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x + \mathrm{cotg}^4 x) =$
$= (1 + \mathrm{cotg}^2 x - \mathrm{cotg}^2 x)[(1 + \mathrm{cotg}^2 x)^2 + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x + \mathrm{cotg}^4 x] =$
$= 1 + 2\,\mathrm{cotg}^2 x + 2\,\mathrm{cotg}^4 x + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x =$
$= 1 + 2\,\mathrm{cotg}^2 x\,(1 + \mathrm{cotg}^2 x) + \mathrm{cossec}^2 x \cdot \mathrm{cotg}^2 x =$
$= 1 + 3\,\mathrm{cotg}^2 x \cdot \mathrm{cossec}^2 x = g(x)$

273. $f(x) = \mathrm{tg}^2\,(45° + x) = \left[\dfrac{\mathrm{tg}\,45° + \mathrm{tg}\,x}{1 - \mathrm{tg}\,45°\,\mathrm{tg}\,x}\right]^2 = \left[\dfrac{\cos x + \mathrm{sen}\,x}{\cos x - \mathrm{sen}\,x}\right]^2 =$
$= \dfrac{1 + 2\,\mathrm{sen}\,x\cos x}{1 - 2\,\mathrm{sen}\,x\cos x} = g(x)$

275. $\dfrac{\pi}{4} < a < \dfrac{\pi}{2} \Rightarrow 0{,}7 < \mathrm{sen}\,a < 1;\ 0 < \cos a < 0{,}71$
$\dfrac{\pi}{4} < b < \dfrac{\pi}{2} \Rightarrow 0{,}7 < \mathrm{sen}\,b < 1;\ 0 < \cos b < 0{,}71$
$\mathrm{sen}\,(a + b) - \mathrm{sen}\,a - \dfrac{4}{5}\,\mathrm{sen}\,b =$
$= \mathrm{sen}\,a\,(\cos b - 1) + \mathrm{sen}\,b\,(\cos a - 0{,}8) < 0 \Rightarrow$
$\Rightarrow \mathrm{sen}\,(a + b) < \mathrm{sen}\,a + \dfrac{4}{5}\,\mathrm{sen}\,b$

276. $f(x) = \left[\mathrm{sen}\,A + \mathrm{sen}\left(\dfrac{\pi}{2} - A\right)\right]^4 = \left[\sqrt{2} \cdot \cos\left(A - \dfrac{\pi}{4}\right)\right]^4 =$
$= 4\cos^4\left(A - \dfrac{\pi}{4}\right) = g(x)$

278. a) $\mathrm{sen}\,B + \mathrm{sen}\,C - \mathrm{sen}\,A = 2\,\mathrm{sen}\,\dfrac{B+C}{2} \cdot \cos\dfrac{B-C}{2} - 2\,\mathrm{sen}\,\dfrac{A}{2} \cdot \cos\dfrac{A}{2} =$
$= 2\cos\dfrac{A}{2} \cdot \cos\dfrac{B-C}{2} - 2\,\mathrm{sen}\,\dfrac{A}{2} \cdot \cos\dfrac{A}{2} =$
$= 2\cos\dfrac{A}{2}\left[\cos\dfrac{B-C}{2} - \cos\dfrac{B+C}{2}\right] =$
$= 2\cos\dfrac{A}{2}\left[-2\,\mathrm{sen}\,\dfrac{B}{2}\,\mathrm{sen}\left(-\dfrac{C}{2}\right)\right] = 4\cos\dfrac{A}{2}\,\mathrm{sen}\,\dfrac{B}{2}\,\mathrm{sen}\,\dfrac{C}{2}$

b) $\cos B + \cos C - \cos A =$

$= 2\cos\frac{B+C}{2}\cos\frac{B-C}{2} - \left(1 - 2\,\text{sen}^2\frac{A}{2}\right) =$

$= -1 + 2\,\text{sen}\frac{A}{2}\left(\cos\frac{B-C}{2} + \text{sen}\frac{A}{2}\right) =$

$= -1 + 2\,\text{sen}\frac{A}{2}\left(\cos\frac{B-C}{2} + \cos\frac{B+C}{2}\right) =$

$= -1 + 2\,\text{sen}\frac{A}{2}\left[2\cos\frac{B}{2}\cdot\cos\left(\frac{-C}{2}\right)\right] =$

$= -1 + 4\cos\frac{B}{2}\cdot\cos\frac{C}{2}\cdot\text{sen}\frac{A}{2}$

c) $\cos 2A + \cos 2B + \cos 2C = 2\cos^2 A - 1 + 2\cos(B+C)\cdot\cos(B-C) =$

$= 2\cos^2 A - 1 + 2(-\cos A)\cos(B-C) =$

$= -1 + 2\cos A\,[\cos A - \cos(B-C)] =$

$= -1 - 2\cos A\,[\cos(B+C) + \cos(B-C)] =$

$= -1 - 2\cos A\,(2\cos B\cos C) = -1 - 4\cos A\cos B\cos C$

d) $\text{sen}^2 A + \text{sen}^2 B + \text{sen}^2 C =$

$= \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + \frac{1-\cos 2C}{2} =$

$= \frac{3 + 1 + 4\cos A\cos B\cos C}{2} = 2(1 + \cos A\cos B\cos C)$

e) $A + B + C = \pi \Rightarrow A + B = \pi - C;$

$\text{cotg}(A+B) = \text{cotg}(\pi - C) = -\text{cotg}\,C \Rightarrow \frac{\text{cotg}\,A\cdot\text{cotg}\,B - 1}{\text{cotg}\,A + \text{cotg}\,B} =$

$= -\text{cotg}\,C \Rightarrow \text{cotg}\,A\cdot\text{cotg}\,B - 1 = -\text{cotg}\,A\cdot\text{cotg}\,C - \text{cotg}\,B\cdot\text{cotg}\,C \Rightarrow$

$\Rightarrow \frac{1}{\text{tg}\,A\cdot\text{tg}\,B} + \frac{1}{\text{tg}\,B\cdot\text{tg}\,C} + \frac{1}{\text{tg}\,C\cdot\text{tg}\,A} = 1$

279. a) $f(a) = \text{sen}\,4a = 2\,\text{sen}\,2a\cdot\cos 2a = 4\,\text{sen}\,a\cdot\cos a\,(\cos^2 a - \text{sen}^2 a) =$

$= 4\,\text{sen}\,a\cdot\cos^3 a - 4\,\text{sen}^3 a\cdot\cos a = g(a)$

b) $f(a) = \cos 4a = 2\cos^2 2a - 1 = 2(2\cos^2 a - 1)^2 - 1 =$

$= 8\cos^4 a - 8\cos^2 a + 1 = g(a)$

c) $f(a) = \frac{2\cdot\text{tg}\,2a}{1 - \text{tg}^2\,2a} = (2\cdot\text{tg}\,2a)\cdot\frac{1}{1-\text{tg}^2\,2a} =$

$= \frac{4\cdot\text{tg}\,a}{1-\text{tg}^2 a}\cdot\frac{1}{1 - \left(\frac{2\cdot\text{tg}\,a}{1-\text{tg}^2 a}\right)^2} =$

$= \frac{4\cdot\text{tg}\,a}{1-\text{tg}^2 a}\cdot\frac{(1-\text{tg}^2 a)^2}{(1-\text{tg}^2 a)^2 - (2\cdot\text{tg}\,a)^2} = \frac{4\cdot\text{tg}\,a - 4\cdot\text{tg}^3 a}{1 - 6\cdot\text{tg}^2 a + \text{tg}^4 a} = g(a)$

280. Prova pelo princípio da indução finita.

Para n = 1, $\frac{\text{sen } 2a}{2 \text{ sen } a} = \frac{2 \text{ sen } a \cos a}{2 \text{ sen } a} = \cos a.$

Admitindo para n = k:

$\cos a \cdot \cos 2a \cdot \ldots \cdot \cos (2^{k-1} \cdot a) = \frac{\text{sen } (2^k \cdot a)}{2^k \cdot \text{sen } a}$

Provemos que vale para n = k + 1:

$1 \cdot \cos a \cdot \cos 2a \cdot \ldots \cdot \cos (2^{k-1} \cdot a) \cdot \cos (2^k \cdot a) \overset{\text{H.I.}}{=}$

$= \frac{\text{sen } (2^k \cdot a)}{2^k \cdot \text{sen } a} \cdot \cos (2^k \cdot a) = \frac{1 \cdot \text{sen } [2 \cdot (2^k \cdot a)]}{2 \cdot 2^k \cdot \text{sen } a} = \frac{\text{sen } (2^{k+1} \cdot a)}{2^{k+1} \cdot \text{sen } a}$

281. a) $\text{cotg } \frac{\alpha}{2} - \text{cotg } \alpha = \frac{1}{\text{tg } \frac{\alpha}{2}} - \frac{1}{\text{tg } \alpha} =$

$= \frac{1}{\text{tg } \frac{\alpha}{2}} - \frac{1 - \text{tg}^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} = \frac{1 + \text{tg}^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} =$

$= \frac{\sec^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} = \frac{1}{2 \cdot \text{sen } \frac{\alpha}{2} \cdot \cos \frac{\alpha}{2}} = \frac{1}{\text{sen } \alpha}$

b) $\frac{1}{\text{sen } a} + \frac{1}{\text{sen } 2a} + \frac{1}{\text{sen } 4a} + \ldots + \frac{1}{\text{sen } (2^n \cdot a)} =$

$= \left(\text{cotg } \frac{a}{2} - \text{cotg } a\right) + (\text{cotg } a - \text{cotg } 2a) + \ldots +$

$+ (\text{cotg } 2^{n-2}a - \text{cotg } 2^{n-1}a) + (\text{cotg } 2^{n-1}a - \text{cotg } 2^n a) =$

$= \text{cotg } \frac{a}{2} - \text{cotg } 2^n a$

282. $\cos^2 x + \text{sen}^2 x + 2 \text{ sen } x \cos x + k \text{ sen } x \cos x - 1 = 0 \Rightarrow$
$\Rightarrow \text{sen } x \cos x \,(2 + k) = 0 \Rightarrow 2 + k = 0 \Rightarrow k = -2$

285. a) $\frac{(-\text{sen } x)(-\text{sen } x)}{-\frac{\text{sen } x}{\cos x}(-\cos x)} = \frac{\text{sen}^2 x}{\text{sen } x} = \text{sen } x$

b) $\frac{\text{sen } x \,(-\text{cotg } x)}{-\text{tg } x \,(-\text{sen } x)} = -\frac{\text{cotg } x}{\text{tg } x} = -\text{cotg}^2 x$

c) $\frac{-\sec x \,(-\text{cotg } x)}{\text{cossec } x \,(-\text{cotg } x)} = -\frac{\text{sen } x}{\cos x} = -\text{tg } x$

d) $-\cos x + \cos x + \text{cotg } x = \text{cotg } x$

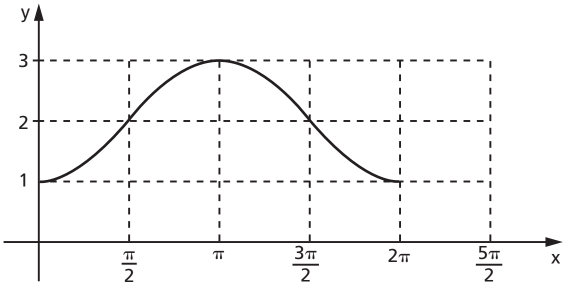
286. $1 - \text{sen } x \cdot \text{sen } x = 1 - \text{sen}^2 x = \cos^2 x$

287. $-1 + \dfrac{(-\text{sen } x) \cdot (-\text{tg } x)}{-\cos x} = -1 - \dfrac{\text{sen}^2 x}{\cos^2 x} = -1 - \text{tg}^2 x = -\sec^2 x$

288. $\dfrac{-a^2 - (a - b)^2 \cdot (-1) + 2ab}{b^2} = \dfrac{b^2}{b^2} = 1$

289. $y = -\cos x + 2$

$\text{Im}(f) = [1, 3];\ p = 2\pi,\ D(f) = \mathbb{R}$



CAPÍTULO XI — Equações

293. $\text{sen}^2 x = t \Rightarrow 4t^2 - 11t + 6 = 0 \Rightarrow t = 2 \text{ ou } t = \dfrac{3}{4}$; $\text{sen}^2 x = 2$ não serve;

$\text{sen}^2 x = \dfrac{3}{4} \Rightarrow \text{sen } x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow x = \pm\dfrac{\pi}{3} + k\pi$

295. a) $5x = 3x + 2k\pi \Rightarrow x = k\pi$

ou

$5x = \pi - 3x + 2k\pi \Rightarrow x = \dfrac{\pi}{8} + \dfrac{k\pi}{4}$

b) $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$

ou

$3x = \pi - 2x + 2k\pi \Rightarrow x = \dfrac{\pi}{5} + 2k\dfrac{\pi}{5}$

296. $\text{sen } x < 0 \Rightarrow 2 \cdot \text{sen } x \cdot (-\text{sen } x) + 3 \cdot \text{sen } x = 2 \Rightarrow$

$\Rightarrow \nexists\, x \in \mathbb{R} \,|\, 2 \text{ sen } x\,|\text{sen } x| + 3 \text{ sen } x - 2 = 0$

$\text{sen } x > 0 \Rightarrow \text{sen } x = t \Rightarrow 2t^2 + 3t - 2 = 0 \Rightarrow t = \dfrac{1}{2} \text{ ou } t = -2;$

$\text{sen } x = -2$ não serve; $\text{sen } x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi \text{ ou } x = \dfrac{5\pi}{6} + 2k\pi$

297. $\text{sen } (x + y) = \text{sen } 0 \Rightarrow \left.\begin{matrix} x + y = k\pi \\ x - y = \pi \end{matrix}\right\} \Rightarrow x = \dfrac{\pi}{2} + \dfrac{k\pi}{2} \text{ e } y = -\dfrac{\pi}{2} + \dfrac{k\pi}{2}$

302. a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \dfrac{k\pi}{2}$

b) $5x = x + \dfrac{\pi}{3} + 2k\pi \Rightarrow x = \dfrac{\pi}{2} + \dfrac{k\pi}{12}$ ou $5x = -x - \dfrac{\pi}{3} + 2k\pi \Rightarrow$

$\Rightarrow x = -\dfrac{\pi}{18} + \dfrac{k\pi}{3}$

303. a) $(\text{sen}\,x + \cos x)(\sec x + \text{cossec}\,x) = 4 \Rightarrow (\text{sen}\,x + \cos x)\left(\dfrac{\text{sen}\,x + \cos x}{\cos x\,\text{sen}\,x}\right) = 4$

$\dfrac{\text{sen}^2 x + 2\,\text{sen}\,x \cos x + \cos^2 x}{\cos x\,\text{sen}\,x} = 4 \Rightarrow \text{sen}\,2x = 1 \Rightarrow$

$\Rightarrow 2x = \dfrac{\pi}{2} + 2k\pi \Rightarrow x = \dfrac{\pi}{4} + k\pi$

b) $\text{sen}\,x = \cos y \Rightarrow \cos^2 x = \text{sen}^2 y \Rightarrow \cos x = \pm\text{sen}\,y$
Notemos que $\cos x = -\text{sen}\,y \Rightarrow \sec x = -\text{cossec}\,y \Rightarrow$
$\Rightarrow (\text{sen}\,x + \cos x)(\sec x + \text{cossec}\,y) = 0 \neq 4$;
então só interessa a hipótese $\cos x = \text{sen}\,y$.
Temos:
$(\text{sen}\,x + \cos y)(\sec x + \text{cossec}\,y) = 4 \Rightarrow$
$\Rightarrow (\text{sen}\,x + \text{sen}\,x)(\sec x + \sec x) = 4 \Rightarrow \text{tg}\,x = 1 \Rightarrow x = \dfrac{\pi}{4} + k\pi$
e daí $y = \dfrac{\pi}{4} + k\pi$.

304. $\text{sen}^2 x = y \Rightarrow y^3 + y^2 + y = 3 \Rightarrow (y - 1)(y^2 + 2y + 3) = 0 \Rightarrow y = 1$

$\text{sen}\,x = \pm 1 \Rightarrow x = \dfrac{\pi}{2} + k\pi \Rightarrow x = (2k + 1)\,\dfrac{\pi}{2}$

305. $2\,\text{sen}\,\dfrac{\pi}{4} \cdot \cos x = \sqrt{2} \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi$

306. $x + y = \pi \Rightarrow x = \pi - y \Rightarrow \text{sen}\,x = \text{sen}\,y$
$\text{sen}\,x + \text{sen}\,y = \log_{10} t^2 \Rightarrow 2\,\text{sen}\,x = 2\log_{10} t \Rightarrow \text{sen}\,x = \log_{10} t$
$-1 \leqslant \log_{10} t \leqslant 1 \Rightarrow 0{,}1 \leqslant t \leqslant 10$

310. a) $1 + \text{tg}^2 x - 2\,\text{tg}\,x = 0$, $\text{tg}\,x = t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1$

$\text{tg}\,x = 1 \Rightarrow x = \dfrac{\pi}{4} + k\pi$

b) $\text{cossec}^2 x = 1 - \text{cotg}\,x \Rightarrow 1 + \text{cotg}^2 x = 1 - \text{cotg}\,x \Rightarrow$
$\Rightarrow \text{cotg}^2 x = \text{cotg}\,x = 0 \Rightarrow \text{cotg}\,x = 0$ ou $\text{cotg}\,x = -1 \Rightarrow$
$\Rightarrow x = \dfrac{\pi}{2} + k\pi$ ou $x = \dfrac{3\pi}{4} + k\pi$

c) $\text{sen}\,2x \cdot \cos\left(x + \dfrac{\pi}{4}\right) - \cos 2x \cdot \text{sen}\left(x + \dfrac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow \text{sen}\left[2x - \left(x + \dfrac{\pi}{4}\right)\right] = 0 \Rightarrow \text{sen}\left(x - \dfrac{\pi}{4}\right) = 0 \Rightarrow x = \dfrac{\pi}{4} + k\pi$

d) $1 + \text{sen } 2x - \text{tg } x - \text{tg } x \text{ sen } 2x = 1 + \text{tg } x \Rightarrow \text{sen } 2x = \dfrac{2 \text{ tg } x}{1 - \text{tg } x} \Rightarrow$

$\Rightarrow \dfrac{2 \text{ tg } x}{1 + \text{tg}^2 x} = \dfrac{2 \text{ tg } x}{1 - \text{tg } x} \Rightarrow \text{tg}^2 x + \text{tg } x = 0 \Rightarrow \text{tg } x (\text{tg } x + 1) = 0 \Rightarrow$

$\Rightarrow \text{tg } x = 0 \Rightarrow x = k\pi$ ou $\text{tg } x = -1 \Rightarrow x = \dfrac{3\pi}{4} + k\pi$

311. $\dfrac{1}{\text{tg } x} = \dfrac{2 \text{ tg } x}{1 + \text{tg}^2 x} \Rightarrow \text{tg}^2 x = 1 \Rightarrow \text{tg } x = \pm 1 \Rightarrow x = \pm\dfrac{\pi}{4} + k\pi$

312. $\text{tg}^2 \dfrac{\pi}{2} p = 1 \Rightarrow \text{tg} \dfrac{\pi}{2} p = \pm 1 \Rightarrow \dfrac{\pi}{2} p = \pm \dfrac{\pi}{4} + k\pi \Rightarrow p = \pm \dfrac{1}{2} + 2k, k \in \mathbb{Z}$

313. $\text{tg } x = 1 \Rightarrow x = \dfrac{\pi}{4} + k\pi$ ou

$\text{sen } x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow x = \pm\dfrac{\pi}{3} + 2k\pi$ ou $x = \pm\dfrac{2\pi}{3} + 2k\pi$;

então as raízes positivas são $\dfrac{\pi}{4}, \dfrac{5\pi}{4}, \ldots, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, \ldots$ e daí

$a = \dfrac{\pi}{4}$ (a menor delas). Temos:

$\text{sen}^4 a - \cos^2 a = \text{sen}^4 \dfrac{\pi}{4} - \cos^2 \dfrac{\pi}{4} = \dfrac{1}{4} - \dfrac{1}{2} = -\dfrac{1}{4}$

314. $\Delta = 4 \text{ tg}^2 a + 4 = 4 \sec^2 a, x = \dfrac{2 \text{ tg } a \pm 2 \sec a}{2} \Rightarrow x = \dfrac{\text{sen } a \pm 1}{\cos a}$

316. a) $\text{sen } x + \text{sen}\left(\dfrac{\pi}{2} - x\right) = -1 \Rightarrow \sqrt{2} \cos\left(x - \dfrac{\pi}{4}\right) = -1 \Rightarrow$

$\Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2} \Rightarrow \left[\cos\left(x - \dfrac{\pi}{4}\right) = \cos\dfrac{3\pi}{4} \Rightarrow\right.$

$\left.\Rightarrow x = \pi + 2k\pi \text{ ou } \cos\left(x - \dfrac{\pi}{4}\right) = \cos\dfrac{5\pi}{4} \Rightarrow x = \dfrac{3\pi}{2} + 2k\pi\right]$

b) $\text{sen } x - \dfrac{1}{\sqrt{3}} \cos x = -1 \Rightarrow \text{sen } x - \dfrac{\text{sen } \dfrac{\pi}{6}}{\cos \dfrac{\pi}{6}} \cdot \cos x = 1 \Rightarrow$

$\Rightarrow \text{sen}\left(x - \dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \left[\text{sen}\left(x - \dfrac{\pi}{6}\right) = \text{sen}\dfrac{4\pi}{3} \Rightarrow x = \dfrac{3\pi}{2} + 2k\pi \text{ ou}\right.$

$\left.\text{sen}\left(x - \dfrac{\pi}{6}\right) = \text{sen}\dfrac{5\pi}{3} \Rightarrow x = \dfrac{11\pi}{6} + 2k\pi\right]$

318. a) sen $4x = u$ e cos $x = v$, $\begin{cases} u + v = 1 \ (1) \\ u^2 + v^2 = 1 \ (2) \end{cases}$, (1) em (2) $\Rightarrow$

$\Rightarrow u^2 + (1 - u)^2 = 1 \Rightarrow 2u^2 - 2u = 0.$

Então: $u = 0$ e $v = 1 \Rightarrow$ sen $4x = 0$ e cos $4x = 1 \Rightarrow x = \frac{k\pi}{2}$

ou $u = 1$ e $v = 0 \Rightarrow$ sen $4x = 1$ e cos $4x = 0 \Rightarrow x = \frac{\pi}{8} + \frac{k\pi}{2}$

b) sen $x = 0$ e cos $x = \pm 1 \Rightarrow x = k\pi$

sen $x = \pm 1$ e cos $x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$

319. sen $2x = u$ e cos $2x = v$, $\begin{cases} u + v = 1 \ (1) \\ u^2 + v^2 = 1 \ (2)^2 \end{cases}$, (1) em (2) $\Rightarrow$

$\Rightarrow u^2 + (1 - u)^2 = 1 \Rightarrow 2u^2 - 2u = 0.$

Então: $u = 0$ e $v = 1 \Rightarrow$ sen $2x = 0$ e cos $2x = 1 \Rightarrow x = k\pi$

ou $u = 1$ e $v = 0 \Rightarrow$ sen $2x = 1$ e cos $2x = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$

321. a) $\operatorname{sen} x = \frac{2t}{1 + t^2}$ e $\cos x = \frac{1 - t^2}{1 + t^2} \Rightarrow m\frac{(1 - t^2)}{1 + t^2} - (m + 1)\frac{2t}{1 + t^2} = m \Rightarrow$

$\Rightarrow t(-2mt - 2m - 2) = 0 \Rightarrow t = 0$ ou $-2mt - 2m - 2 = 0 \Rightarrow$

$\Rightarrow t = \frac{-(m + 1)}{m}$, $m \neq 0$

$m = 0 \Rightarrow 0 \cdot \cos x - \operatorname{sen} x = 0, \exists x, \forall m \in \mathbb{R}.$

b) sen $x = \frac{2t}{1 + t^2}$ e cos $x = \frac{1 - t^2}{1 + t^2} \Rightarrow \frac{2t}{1 + t^2} + \frac{1 - t^2}{1 + t^2} = m \Rightarrow$

$\Rightarrow (-m - 1)t^2 + 2t + 1 - m = 0 \Rightarrow \Delta = -4m^2 + 8 \geqslant 0 \Rightarrow$

$\Rightarrow -\sqrt{2} \leqslant m \leqslant \sqrt{2}$

323. a) $2 \operatorname{sen}\left(\frac{mx + nx}{2}\right) \cdot \cos\left(\frac{mx - nx}{2}\right) = 0$. Então: $\operatorname{sen}\frac{(m + n)x}{2} = 0 \Rightarrow$

$\Rightarrow x = \frac{2k\pi}{m + n}$

ou $\cos\frac{(m - n)x}{2} = 0 \Rightarrow x = \frac{\pi}{m - n} + \frac{2k\pi}{m - n}$

b) $2 \cos\frac{ax + bx}{2} \cdot \cos\frac{ax - bx}{2} = 0$. Então: $\cos\frac{(a + b)x}{2} = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{a + b} + \frac{2k\pi}{a + b}$

ou $\cos\frac{(a - b)x}{2} = 0 \Rightarrow x = \frac{\pi}{a - b} + \frac{2k\pi}{a - b}$

c) $\text{sen } 2x - \text{sen}\left(\frac{\pi}{4} - x\right) = 0 \Rightarrow 2 \text{ sen}\left(\frac{3x}{2} - \frac{\pi}{8}\right)\cos\left(\frac{x}{2} + \frac{\pi}{8}\right) = 0$

Então:

$\text{sen}\left(\frac{3x}{2} - \frac{\pi}{8}\right) = 0 \Rightarrow x = \frac{\pi}{12} + \frac{2k\pi}{3}$ ou $\cos\left(\frac{x}{2} + \frac{\pi}{8}\right) = 0 \Rightarrow$

$\Rightarrow x = \frac{3\pi}{4} + 2k\pi$

325. a) $2 \text{ sen } 3x \cdot \cos 2x - 2 \text{ sen } 3x = 0 \Rightarrow 2 \text{ sen } 3x (\cos 2x - 1) = 0.$

Então: $\text{sen } 3x = 0 \Rightarrow x = \frac{k\pi}{3}$ ou $\cos 2x = 1 \Rightarrow x = k\pi.$

b) $2 \cos (2x + a) \cos (x + a) + \cos (2x + a) = 0 \Rightarrow$

$\Rightarrow \cos (2x + a)[2 \cos (x + a) + 1] = 0.$ Então: $\cos (2x + a) = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} - \frac{a}{2} + \frac{k\pi}{2}$

ou $2 \cos (x + a) + 1 = 0 \Rightarrow \cos (x + a) = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} - a + 2k\pi$

ou $x = \frac{4\pi}{3} - a + 2k\pi$

c) $2 \text{ sen } 4x \cdot \cos 3x - 2 \text{ sen } 4x \cdot \text{sen }(-x) = 0 \Rightarrow$

$\Rightarrow 2 \text{ sen } 4x (\cos 3x + \text{sen } x) = 0 \Rightarrow$

$\Rightarrow 2 \text{ sen } 4x \left[\cos 3x + \cos\left(\frac{\pi}{2} - x\right)\right] = 0 \Rightarrow$

$\Rightarrow 4 \text{ sen } 4x \cdot \cos\left(x + \frac{\pi}{4}\right) \cdot \cos\left(2x - \frac{\pi}{4}\right) = 0$

Então:

$\text{sen } 4x = 0 \Rightarrow x = \frac{k\pi}{4}$ ou $\cos\left(x + \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} + k\pi$ ou $\cos\left(2x - \frac{\pi}{4}\right) = 0 \Rightarrow x = -\frac{3\pi}{8} + \frac{k\pi}{2}$

326. $\frac{1 + \cos (2x + 2a)}{2} + \frac{1 + \cos (2x - 2a)}{2} = 1 \Rightarrow$

$\Rightarrow \cos (2x + 2a) + \cos (2x - 2a) = 0 \Rightarrow 2 \cos (2x) \cdot \cos (2a) = 0 \Rightarrow$

$\Rightarrow \cos (2x) = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$ ou $x = \frac{3\pi}{4} + k\pi$ (supondo $\cos 2a \neq 0$)

327. $(\text{sen } 3x - \text{sen } x) + (\cos 2x - \cos 0) = 0 \Rightarrow$

$\Rightarrow 2 \cdot \text{sen } x \cdot \cos 2x - 2 \cdot \text{sen}^2 x = 0 \Rightarrow 2 \cdot \text{sen } x \cdot (\cos 2x - \text{sen } x) = 0$

Então:

$\text{sen } x = 0 \Rightarrow x = k\pi$ ou

$\cos 2x = \text{sen } x \Rightarrow \cos 2x = \cos\left(\frac{\pi}{2} - x\right) \Rightarrow 2x = \pm\left(\frac{\pi}{2} - x\right) + 2k\pi \Rightarrow$

$\Rightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3}$ ou $x = -\frac{\pi}{2} + 2k\pi$

328. $2 \cdot \text{sen}\, x \cdot \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \text{sen}\, x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

329. a) Substituindo $x = 2\pi$ e $y = \frac{\pi}{2}$ na equação (1):

$\text{sen}\left(2\pi + \frac{\pi}{2}\right) + \text{sen}\left(2\pi - \frac{\pi}{2}\right) = 1 - 1 \neq 2.$

b) $\left.\begin{array}{l} 2 \cdot \text{sen}\, x \cdot \cos y = 2 \\ \text{sen}\, x + \cos y = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \text{sen}\, x \cdot \cos y = 1 \text{ (A)} \\ \text{sen}\, x + \cos y = 2 \text{ (B)} \end{array}\right\}$, (A) em (B) $\Rightarrow$

$\Rightarrow \text{sen}^2\, x - 2\,\text{sen}\, x + 1 = 0$

Então:

$\text{sen}\, x = 1$ (C) $\Rightarrow x = \frac{\pi}{2} + 2k\pi$, (C) em (A) $\Rightarrow \cos y = 1 \Rightarrow y = 2k\pi$

330. $(\cos x + 1)(\text{sen}\, x + 1) = 0$. Então: $\cos x + 1 = 0 \Rightarrow x = \pi + 2k\pi$ ou $\text{sen}\, x + 1 = 0 \Rightarrow x = \frac{3\pi}{2} + 2k\pi$

333. a) $(\cos^2 x + \text{sen}^2\, x)^2 - 2 \cdot \text{sen}^2\, x \cdot \cos^2 x = \frac{5}{8} \Rightarrow$

$\Rightarrow 1 - 2 \cdot \text{sen}^2\, x \cdot \cos^2 x = \frac{5}{8} \Rightarrow 1 - \frac{1}{2}\,\text{sen}^2\, 2x = \frac{5}{8} \Rightarrow$

$\Rightarrow \text{sen}^2\, 2x = \frac{3}{4}$

Então: $\text{sen}\, 2x = \frac{+\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} + k\pi$ ou $x = \frac{\pi}{3} + k\pi$;

$\text{sen}\, 2x = \frac{-\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3} + k\pi$ ou $x = \frac{5\pi}{6} + k\pi$

b) $(\cos^2 x + \text{sen}^2\, x)(\cos^4 x - \cos^2 x \cdot \text{sen}^2\, x + \text{sen}^4\, x) = \frac{5}{8} \Rightarrow$

$\Rightarrow (\text{sen}^4\, x + \cos^4 x) - \text{sen}^2\, x \cdot \cos^2 x = \frac{5}{8} \Rightarrow$

$\Rightarrow \left(1 - \frac{1}{2} \cdot \text{sen}^2\, 2x\right) - \frac{1}{4} \cdot \text{sen}^2\, 2x = \frac{5}{8} \Rightarrow 1 - \frac{3}{4} \cdot \text{sen}^2\, 2x = \frac{5}{8}$

Então: $\text{sen}\, 2x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{8} + k\pi$ ou $x = \frac{3\pi}{8} + k\pi$ ou

$\text{sen}\, 2x = \frac{-\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{8} + k\pi$ ou $x = \frac{7\pi}{8} + k\pi$

c) $1 - \frac{1}{2} \cdot \text{sen}^2\, 2x = \frac{1}{2}$. Então: $\text{sen}\, 2x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

ou $\text{sen}\, 2x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$

d) $1 - \frac{3}{4} \cdot \text{sen}^2\, x = \frac{7}{16}$. Então: $\text{sen}\, x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} + k\pi$ ou

$x = \frac{2\pi}{3} + k\pi$

e) $(\text{sen } x + \cos x)(\text{sen}^2 x + \cos^2 x - \text{sen } x \cdot \cos x) = 1 \Rightarrow$

$\Rightarrow (\text{sen } x + \cos x)(1 - \text{sen } x \cdot \cos x) = 1$. Fazendo $\text{sen } x + \cos x = y$, temos $(\text{sen } x + \cos x)^2 = y^2$ e daí: $\text{sen } x \cdot \cos x = \dfrac{y^2 - 1}{2}$.

A equação fica $y \cdot \left(1 - \dfrac{y^2 - 1}{2}\right) = 1 \Rightarrow (y - 1)^2(y + 2) = 0 \Rightarrow$

$\Rightarrow y = 1$ ou $y = -2$ não serve, pois $-\sqrt{2} \leqslant y \leqslant \sqrt{2}$; então $\text{sen } x + \cos x = 1 \Rightarrow$

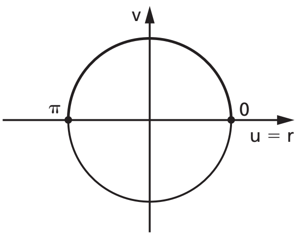
$$\Rightarrow \sqrt{2} \cdot \cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow x = \frac{\pi}{2} + 2k\pi \text{ ou } x = 2k\pi$$

CAPÍTULO XII — Inequações

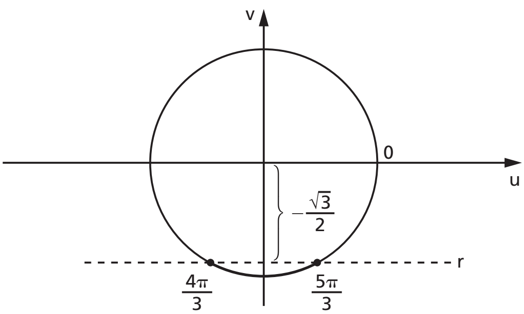
335. $\text{sen } x = 0 \Rightarrow x = 2k\pi$ ou $x = \pi + 2k\pi$

$S = \{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \pi + 2k\pi\}$



336. $\text{sen } x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{4\pi}{3} + 2k\pi$ ou $x = \dfrac{5\pi}{3} + 2k\pi$

$$S = \left\{x \in \mathbb{R} \mid \frac{4\pi}{3} + 2k\pi \leqslant x \leqslant \frac{5\pi}{3} + 2k\pi\right\}$$

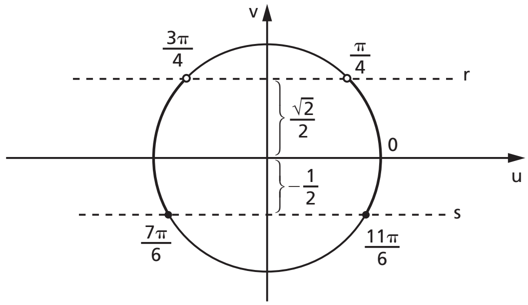


337. $\text{sen } x = \dfrac{\sqrt{2}}{2} \Rightarrow x = \dfrac{\pi}{4} + 2k\pi \text{ ou } x = \dfrac{3\pi}{4} + 2k\pi$

$\text{sen } x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6} + 2k\pi \text{ ou } x = \dfrac{11\pi}{6} + 2k\pi$

$S = \left\{x \in \mathbb{R} \mid 2k\pi \leqslant x < \dfrac{\pi}{4} + 2k\pi \text{ ou } \dfrac{3\pi}{4} + 2k\pi < x \leqslant \dfrac{7\pi}{6} + 2k\pi \right.$

$\left. \text{ou } \dfrac{11\pi}{6} + 2k\pi \leqslant x < 2\pi + 2k\pi \right\}$



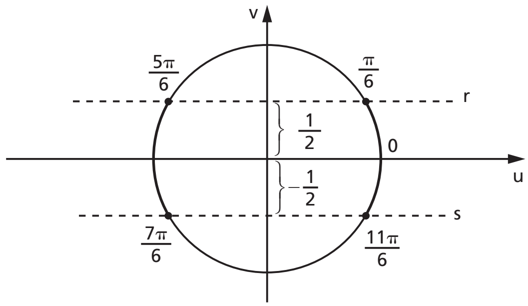
339. $\text{sen } x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi \text{ ou } x = \dfrac{5\pi}{6} + 2k\pi$

$\text{sen } x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6} + 2k\pi \text{ ou } x = \dfrac{11\pi}{6} + 2k\pi$

$|\text{sen } x| \leqslant \dfrac{1}{2} \Leftrightarrow -\dfrac{1}{2} \leqslant \text{sen } x \leqslant \dfrac{1}{2}$

$S = \left\{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \dfrac{\pi}{6} + 2k\pi \text{ ou } \dfrac{5\pi}{6} + 2k\pi \leqslant x \leqslant \dfrac{7\pi}{6} + 2k\pi \right.$

$\left. \text{ou } \dfrac{11\pi}{6} + 2k\pi \leqslant x < 2\pi + 2k\pi \right\}$

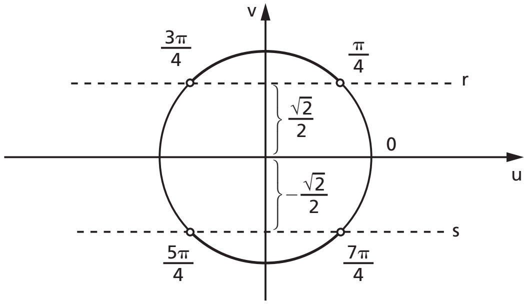


340. $|\text{sen } x| > \dfrac{\sqrt{2}}{2} \Leftrightarrow \text{sen } x > \dfrac{\sqrt{2}}{2}$ ou $\text{sen } x < -\dfrac{\sqrt{2}}{2}$

$\text{sen } x = \dfrac{\sqrt{2}}{2} \Rightarrow x = \dfrac{\pi}{4} + 2k\pi$ ou $x = \dfrac{3\pi}{4} + 2k\pi$

$\text{sen } x = -\dfrac{\sqrt{2}}{2} \Rightarrow x = \dfrac{5\pi}{4} + 2k\pi$ ou $x = \dfrac{7\pi}{4} + 2k\pi$

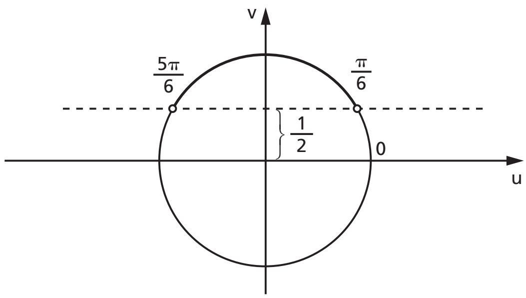
$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{4} + 2k\pi < x < \dfrac{3\pi}{4} + 2k\pi \text{ ou } \dfrac{5\pi}{4} + 2k\pi < x < \dfrac{7\pi}{4} + 2k\pi\right\}$



342. a) $2 \text{ sen } x - 1 > 0 \Rightarrow \text{sen } x > \dfrac{1}{2}$

$\text{sen } x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} + 2k\pi < x < \dfrac{5\pi}{6} + 2k\pi\right\}$



b) $2 \cdot \log_2 (2 \cdot \text{sen } x - 1) = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

$\Rightarrow \log_2 (2 \cdot \text{sen } x - 1)^2 = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

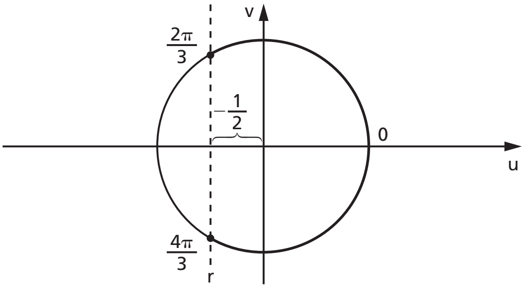
$\Rightarrow (2 \cdot \text{sen } x - 1)^2 = (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2) \Rightarrow$

$\Rightarrow \text{sen}^2 x = 1 \Rightarrow \text{sen } x = \pm 1$

Só convém $\text{sen } x = 1$ (devido à parte a), então $x = \dfrac{\pi}{2} + 2k\pi$.

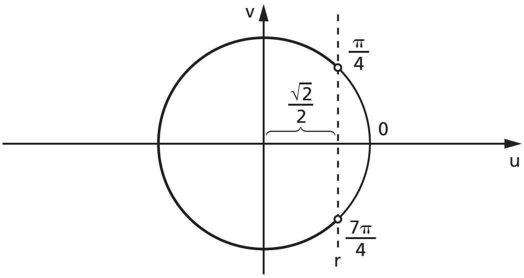
344. $\cos x = -\dfrac{1}{2} \Rightarrow x = \dfrac{2\pi}{3} + 2k\pi$ ou $x = \dfrac{4\pi}{3} + 2k\pi$

$S = \left\{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \dfrac{2\pi}{3} + 2k\pi \text{ ou } \dfrac{4\pi}{3} + 2k\pi \leqslant x < 2\pi + 2k\pi\right\}$



345. $\cos x = \dfrac{\sqrt{2}}{2} \Rightarrow x = \dfrac{\pi}{4} + 2k\pi$ ou $x = \dfrac{7\pi}{4} + 2k\pi$

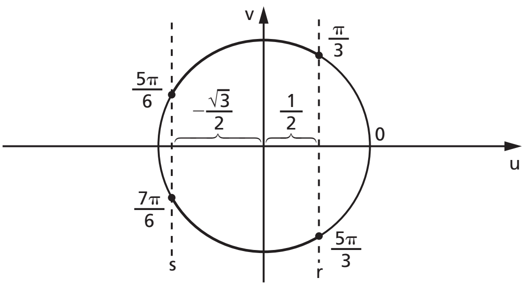
$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{4} + 2k\pi < x < \dfrac{7\pi}{4} + 2k\pi\right\}$



346. $\cos x = \dfrac{1}{2} \Rightarrow x = \dfrac{\pi}{3} + 2k\pi$ ou $x = \dfrac{5\pi}{3} + 2k\pi$

$\cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{5\pi}{6} + 2k\pi$ ou $x = \dfrac{7\pi}{6} + 2k\pi$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{3} + 2k\pi \leqslant x \leqslant \dfrac{5\pi}{6} + 2k\pi \text{ ou } \dfrac{7\pi}{6} + 2k\pi \leqslant x \leqslant \dfrac{5\pi}{3} + 2k\pi\right\}$

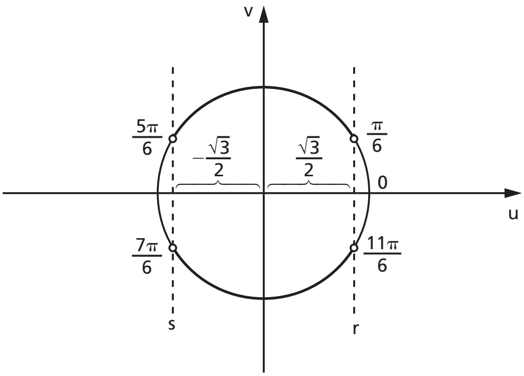


347. $|\cos x| < \dfrac{\sqrt{3}}{2} \Rightarrow -\dfrac{\sqrt{3}}{2} < \cos x < \dfrac{\sqrt{3}}{2}$

$\cos x = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{11\pi}{6} + 2k\pi$

$\cos x = -\dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{5\pi}{6} + 2k\pi$ ou $x = \dfrac{7\pi}{6} + 2k\pi$

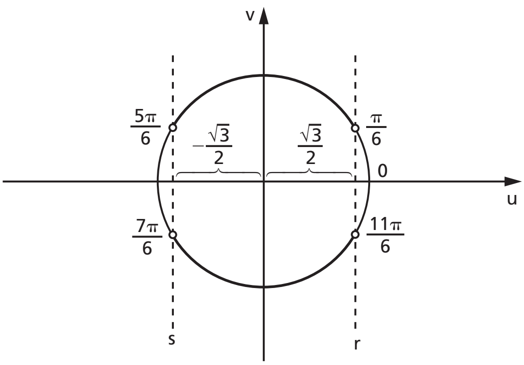
$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} + 2k\pi < x < \dfrac{5\pi}{6} + 2k\pi \text{ ou } \dfrac{7\pi}{6} + 2k\pi < x < \dfrac{11\pi}{6} + 2k\pi\right\}$



348. $|\cos x| > \dfrac{5}{3}$; impossível, pois $-1 \leqslant \cos x \leqslant 1$; $S = \varnothing$

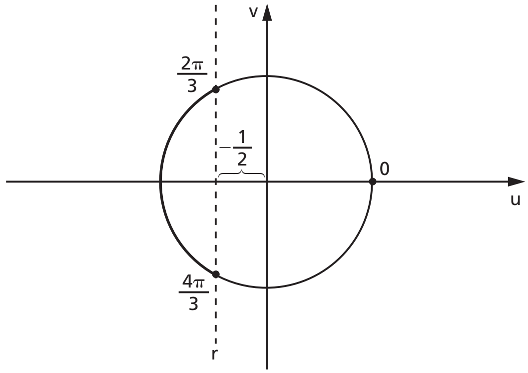
350. $\cos^2 x < \dfrac{3}{4} \Leftrightarrow -\dfrac{\sqrt{3}}{2} < \cos x < \dfrac{\sqrt{3}}{2}$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} + 2k\pi < x < \dfrac{5\pi}{6} + 2k\pi \text{ ou } \dfrac{7\pi}{6} + 2k\pi < x < \dfrac{11\pi}{6} + 2k\pi\right\}$



351. $2\cos^2 x - 1 - \cos x \geqslant 0 \Leftrightarrow \cos x \leqslant -\dfrac{1}{2}$ ou $\cos x = 1$

$$S = \left\{x \in \mathbb{R} \mid \frac{2\pi}{3} + 2k\pi \leqslant x \leqslant \frac{4\pi}{3} + 2k\pi \text{ ou } x = 2k\pi\right\}$$



353. $\operatorname{sen} x + \operatorname{sen}\left(\dfrac{\pi}{2} - x\right) < 1 \Rightarrow \sqrt{2}\cos\left(x - \dfrac{\pi}{4}\right) < 1 \Rightarrow$

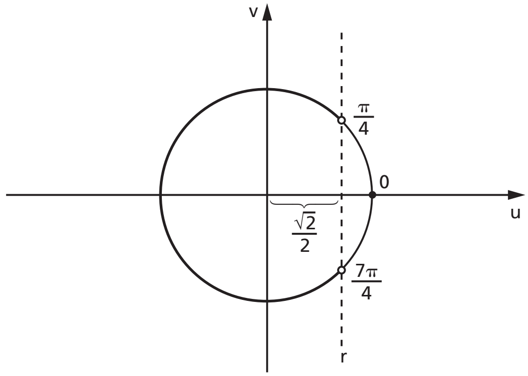
$\Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) < \dfrac{\sqrt{2}}{2}$

Fazendo $x - \dfrac{\pi}{4} = y$, temos:

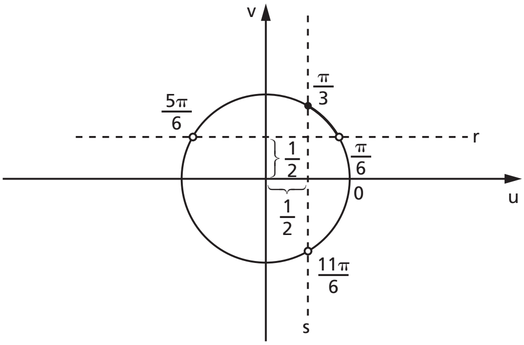
$\dfrac{\pi}{4} + 2k\pi < y < \dfrac{7\pi}{4} + 2k\pi$;

então $\dfrac{\pi}{4} + 2k\pi < x - \dfrac{\pi}{4} < \dfrac{7\pi}{4} + 2k\pi$

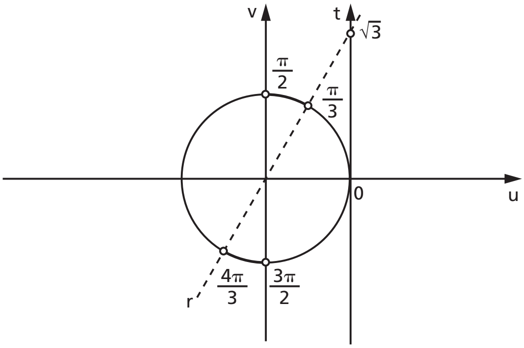
$$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{2} + 2k\pi < x < 2\pi + 2k\pi\right\}$$



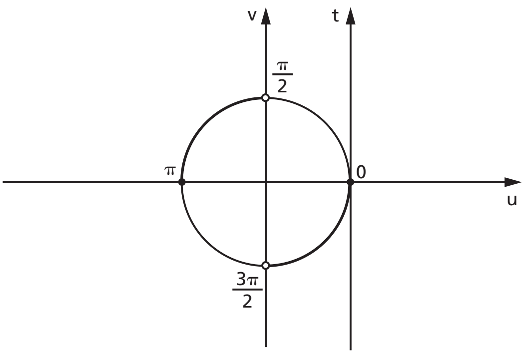
355. $S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} + 2k\pi < x \leqslant \dfrac{\pi}{3} + 2k\pi\right\}$



357. $S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{3} + k\pi < x < \dfrac{\pi}{2} + k\pi\right\}$

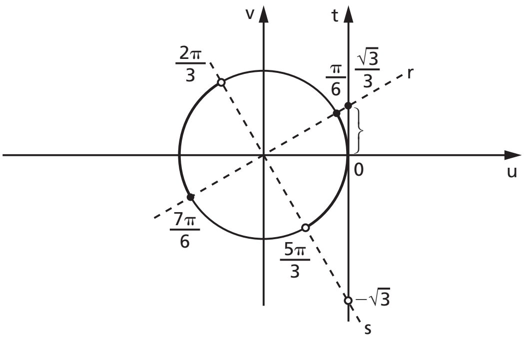


358. $S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{2} + k\pi < x \leqslant \pi + k\pi\right\}$



359. $S = \left\{x \in \mathbb{R} \mid 2k\pi \leqslant x \leqslant \frac{\pi}{6} + 2k\pi \text{ ou } \right.$

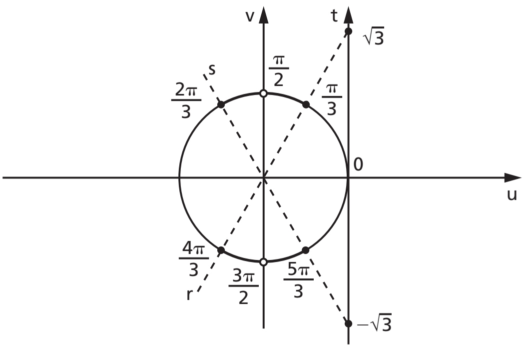
$\left. \frac{2\pi}{3} + 2k\pi < x \leqslant \frac{7\pi}{6} + 2k\pi \text{ ou } \frac{5\pi}{3} + 2k\pi < x \leqslant 2\pi + 2k\pi \right\}$



360. $\text{tg } x \leqslant -\sqrt{3} \text{ ou tg } x \geqslant \sqrt{3}$

$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{3} + k\pi \leqslant x < \frac{\pi}{2} + k\pi \text{ ou } \right.$

$\left. \frac{\pi}{2} + k\pi < x \leqslant \frac{2\pi}{3} + k\pi \right\}$



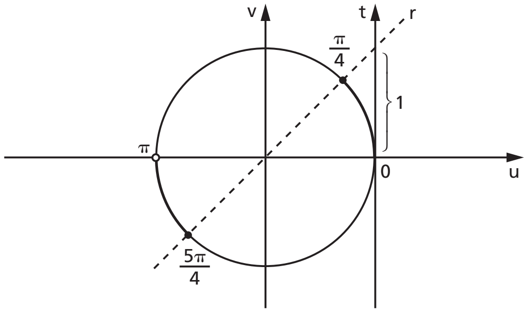
361. C.E. $\text{tg } x > 0$ (A)

$\log y = \log a^{\log \text{tg } x} \geqslant 0 \Rightarrow \log a^{\log \text{tg } x} \geqslant \log 1 \Rightarrow \log a^{\log \text{tg } x} \geqslant \log a^0 \Rightarrow$

$\Rightarrow a^{\log \text{tg } x} \geqslant a^0 \Rightarrow \log \text{tg } x \leqslant 0 \Rightarrow \log \text{tg } x \leqslant \log 1 \Rightarrow \text{tg } x \leqslant 1$ (B)

De (A) e (B) $\Rightarrow 0 < \text{tg } x \leqslant 1$

$S = \left\{x \in \mathbb{R} \mid 0 < x \leqslant \frac{\pi}{4} \text{ ou } \pi < x \leqslant \frac{5\pi}{4}\right\}$



CAPÍTULO XIII — Funções circulares inversas

363. a) $\alpha = \text{arc sen } 0 \Leftrightarrow \text{sen } \alpha = 0 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = 0$

b) $\alpha = \text{arc sen } \frac{\sqrt{3}}{2} \Leftrightarrow \text{sen } \alpha = \frac{\sqrt{3}}{2} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{3}$

c) $\alpha = \text{arc sen}\left(-\frac{1}{2}\right) \Leftrightarrow \text{sen } \alpha = -\frac{1}{2} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{6}$

d) $\alpha = \text{arc sen } 1 \Leftrightarrow \text{sen } \alpha = 1 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2}$

e) $\alpha = \text{arc sen } (-1) \Leftrightarrow \text{sen } \alpha = -1 \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{2}$

367. a) $\alpha = \text{arc sen}\left(-\frac{2}{3}\right) \Rightarrow \text{sen } \alpha = -\frac{2}{3}, -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

$\beta = \text{arc sen } \frac{1}{4} \Rightarrow \text{sen } \beta = \frac{1}{4}, -\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2}$

$\text{sen}^2 \alpha = \frac{\text{tg}^2 \alpha}{1 + \text{tg}^2 \alpha} \Rightarrow \text{tg } \alpha = \frac{-2\sqrt{5}}{5}; \text{ sen}^2 \beta = \frac{\text{tg}^2 \beta}{1 + \text{tg}^2 \beta} \Rightarrow$

$\Rightarrow \text{tg } \beta = \frac{\sqrt{15}}{15}$

$\text{tg } (\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \cdot \text{tg } \beta} = \frac{\sqrt{5}\,(-6 + \sqrt{3})}{15 + 2\sqrt{3}}$

b) $\alpha = \text{arc sen}\left(-\frac{3}{5}\right) \Rightarrow \text{sen } \alpha = -\frac{3}{5} \text{ e } -\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$

$\cos \alpha = \sqrt{1 - \text{sen}^2 \alpha} = \frac{4}{5}; \text{ sen } 2\alpha = 2 \cdot \text{sen } \alpha \cdot \cos \alpha = -\frac{24}{25}$

c) $\beta = \text{arc sen } \frac{12}{13} \Rightarrow \text{sen } \beta = \frac{12}{13} \text{ e } -\frac{\pi}{2} \leqslant \beta \leqslant \frac{\pi}{2};$

$\cos \beta = \sqrt{1 - \text{sen}^2 \beta} = \frac{5}{13}; \cos 3\beta = 4\cos^3 \beta - 3\cos \beta = -\frac{2\,035}{2\,197}$

368. $\text{arc sen}\ \dfrac{1}{2} = \alpha \Rightarrow \text{sen}\ \alpha = \dfrac{1}{2}\ \text{e}\ -\dfrac{\pi}{2} \leqslant \alpha \leqslant \dfrac{\pi}{2} \Rightarrow \alpha = \dfrac{\pi}{6}$

$\text{arc sen}\ x = 2 \cdot \dfrac{\pi}{6} \Rightarrow \text{arc sen}\ x = \dfrac{\pi}{3} \Rightarrow \text{sen}\ \dfrac{\pi}{3} = x \Rightarrow x = \dfrac{\sqrt{3}}{2}$

370. a) $\beta = \text{arc cos}\ 1 \Rightarrow \cos \beta = 1\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow \beta = 0$

b) $\beta = \text{arc cos}\ \dfrac{1}{2} \Rightarrow \cos \beta = \dfrac{1}{2}\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \dfrac{\pi}{3}$

c) $\beta = \text{arc cos}\ \dfrac{\sqrt{2}}{2} \Rightarrow \cos \beta = \dfrac{\sqrt{2}}{2}\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \dfrac{\pi}{4}$

d) $\beta = \text{arc cos}\ 0 \Rightarrow \cos \beta = 0\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \dfrac{\pi}{2}$

e) $\beta = \text{arc cos}\ (-1) \Rightarrow \cos \beta = -1\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow \beta = \pi$

372. $\beta = \text{arc cos}\left(-\dfrac{3}{5}\right) \Rightarrow \cos \beta = -\dfrac{3}{5}\ \text{e}\ 0 \leqslant \beta \leqslant \pi$

$\text{sen}\ \beta = \sqrt{1 - \cos^2 \beta} \Rightarrow \text{sen}\ \beta = \dfrac{4}{5}$

373. $\beta = \text{arc cos}\ \dfrac{2}{7} \Rightarrow \cos \beta = \dfrac{2}{7}\ \text{e}\ 0 \leqslant \beta \leqslant \pi;\ \text{sen}\ \beta = \sqrt{1 - \cos^2 \beta} =$

$= \dfrac{3\sqrt{5}}{7}$

$\text{cotg}\ \beta = \dfrac{\cos \beta}{\text{sen}\ \beta} = \dfrac{2\sqrt{5}}{15}$

374. $\left.\begin{array}{l}\text{arc sen}\ x = A \Rightarrow \text{sen}\ A = x \\ \text{arc cos}\ x = B \Rightarrow \cos B = x\end{array}\right\} \Rightarrow \text{sen}\ A = \cos B \Rightarrow B = \dfrac{\pi}{2} - A = \text{arc cos}\ x$

376. a) $\text{arc cos}\ \dfrac{3}{5} = \beta \Rightarrow \cos \beta = \dfrac{3}{5}\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow$

$\Rightarrow \text{sen}\ \beta = \sqrt{1 - \cos^2 \beta} = \dfrac{4}{5}$

$\text{arc cos}\ \dfrac{5}{13} = \alpha \Rightarrow \cos \alpha = \dfrac{5}{13}\ \text{e}\ 0 \leqslant \alpha \leqslant \pi \Rightarrow$

$\Rightarrow \text{sen}\ \alpha = \sqrt{1 - \cos^2 \alpha} = \dfrac{12}{13}$

$\text{sen}\ (\beta - \alpha) = \text{sen}\ \beta \cdot \cos \alpha - \text{sen}\ \alpha \cdot \cos \beta = -\dfrac{16}{65}$

b) $\text{arc sen}\ \dfrac{7}{25} = \beta \Rightarrow \text{sen}\ \beta = \dfrac{7}{25}\ \text{e}\ -\dfrac{\pi}{2} \leqslant \beta \leqslant \dfrac{\pi}{2} \Rightarrow \cos \beta = \dfrac{24}{25}$

$\text{arc cos}\ \dfrac{12}{13} = \alpha \Rightarrow \cos \alpha = \dfrac{12}{13}\ \text{e}\ 0 \leqslant \alpha \leqslant \pi \Rightarrow \text{sen}\ \alpha = \dfrac{5}{13}$

$\cos (\beta - \alpha) = \cos \beta \cdot \cos \alpha + \text{sen}\ \beta \cdot \text{sen}\ \alpha = \dfrac{323}{325}$

c) $\text{arc cos}\left(-\dfrac{3}{5}\right) = \beta \Rightarrow \cos\beta = -\dfrac{3}{5}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow \text{sen}\,\beta = \dfrac{4}{5}$ e

$\text{tg}\,\beta = \dfrac{\text{sen}\,\beta}{\cos\beta} = -\dfrac{4}{3} \Rightarrow \text{tg}\,2\beta = \dfrac{2\,\text{tg}\,\beta}{1 - \text{tg}^2\,\beta} = \dfrac{24}{7}$

d) $\text{arc cos}\,\dfrac{7}{25} = \beta \Rightarrow \cos\beta = \dfrac{7}{25}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow$

$\Rightarrow \cos\dfrac{\beta}{2} = \sqrt{\dfrac{1 + \cos\beta}{2}} = \dfrac{4}{5}$

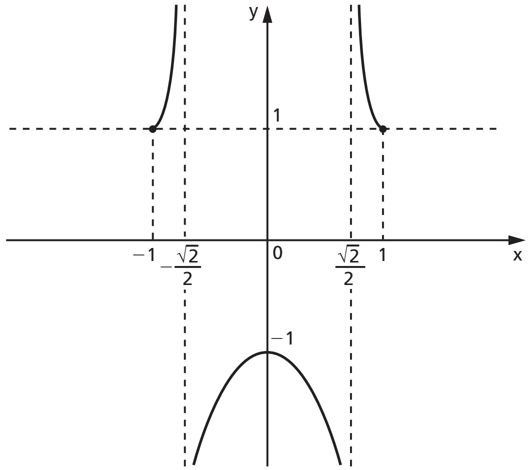
377. a) $\cos(2\,\text{arc cos}\,x) = 0$

$\text{arc cos}\,x = \beta \Rightarrow \cos\beta = x$ e $0 \leqslant \beta \leqslant \pi$

$\cos 2\beta = 0 \Rightarrow 2\cos^2\beta - 1 = 0 \Rightarrow 2x^2 - 1 = 0 \Rightarrow x = \pm\dfrac{\sqrt{2}}{2}$

b) $g(x) = \dfrac{1}{f(x)} = \dfrac{1}{\cos 2\beta} = \dfrac{1}{2x^2 - 1}$, $-1 \leqslant x \leqslant 1$ e $x \neq +\dfrac{\sqrt{2}}{2}$ e

$x \neq -\dfrac{\sqrt{2}}{2}$



378. $\beta = \text{arc cos}\,\sqrt{2} \Rightarrow \cos\beta = \sqrt{2}$, $\nexists\,\beta$ pois $-1 \leqslant \cos\beta \leqslant 1$, $\forall\,\beta \in \mathbb{R}$

380. $\text{arc tg}\,0 = \alpha \Rightarrow \text{tg}\,\alpha = 0$ e $-\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2} \Rightarrow \alpha = 0$

$\text{arc tg}\,\sqrt{3} = \beta \Rightarrow \text{tg}\,\beta = \sqrt{3}$ e $-\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow \beta = \dfrac{\pi}{3}$

$\text{arc tg}\,(-1) = \alpha \Rightarrow \text{tg}\,\alpha = -1$ e $-\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2} \Rightarrow \alpha = -\dfrac{\pi}{4}$

$\text{arc tg}\left(-\dfrac{\sqrt{3}}{3}\right) = \beta \Rightarrow \text{tg}\,\beta = -\dfrac{\sqrt{3}}{3}$ e $-\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow \beta = -\dfrac{\pi}{6}$

382. $\text{arc tg}\left(-\dfrac{4}{3}\right) = \beta \Rightarrow \text{tg}\,\beta = -\dfrac{4}{3} \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2}$

$\cos\beta = \sqrt{\dfrac{1}{1 + \text{tg}^2\,\beta}} = \dfrac{3}{5}$

384. a) $\text{arc tg } 2 = \alpha \Rightarrow \text{tg}\,\alpha = 2 \text{ e } -\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2}$

$\text{arc tg } 3 = \beta \Rightarrow \text{tg}\,\beta = 3 \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2}$

$\cos\alpha = \sqrt{\dfrac{1}{1 + \text{tg}^2\,\alpha}} = \dfrac{\sqrt{5}}{5}$; $\cos\beta = \sqrt{\dfrac{1}{1 + \text{tg}^2\,\beta}} = \dfrac{\sqrt{10}}{10}$

$\text{sen}\,\alpha = \sqrt{\dfrac{\text{tg}^2\,\alpha}{1 + \text{tg}^2\,\alpha}} = \dfrac{2\sqrt{5}}{5}$; $\text{sen}\,\beta = \sqrt{\dfrac{\text{tg}^2\,\beta}{1 + \text{tg}^2\,\beta}} = \dfrac{3\sqrt{10}}{10}$

$\text{sen}\,(\alpha + \beta) = \text{sen}\,\alpha \cdot \cos\beta + \text{sen}\,\beta \cdot \cos\alpha = \dfrac{\sqrt{2}}{2}$

b) $\text{arc tg } 2 = \beta \Rightarrow \text{tg}\,\beta = 2 \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow$

$\Rightarrow \text{sen}\,\beta = \dfrac{2\sqrt{5}}{5}, \cos\beta = \dfrac{\sqrt{5}}{5}$

$\text{arc tg}\,\dfrac{1}{2} = \gamma \Rightarrow \text{tg}\,\gamma = \dfrac{1}{2} \text{ e } -\dfrac{\pi}{2} \leqslant \gamma \leqslant \dfrac{\pi}{2} \Rightarrow$

$\Rightarrow \text{sen}\,\gamma = \dfrac{\sqrt{5}}{5}, \cos\gamma = \dfrac{2\sqrt{5}}{5}$

$\cos(\beta - \gamma) = \cos\beta \cdot \cos\gamma + \text{sen}\,\beta \cdot \text{sen}\,\gamma = \dfrac{4}{5}$

c) $\text{arc tg}\,\dfrac{1}{5} = \beta \Rightarrow \text{tg}\,\beta = \dfrac{1}{5} \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow$

$\Rightarrow \text{tg}\,2\beta = \dfrac{2\,\text{tg}\,\beta}{1 - \text{tg}^2\,\beta} = \dfrac{5}{12}$

d) $\text{arc tg}\,\dfrac{24}{7} = \beta \Rightarrow \text{tg}\,\beta = \dfrac{24}{7} \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow$

$\Rightarrow \cos\beta = \sqrt{\dfrac{1}{1 + \text{tg}^2\,\beta}} = \dfrac{7}{25}$

$\cos 3\beta = 4 \cdot \cos^3\beta - 3 \cdot \cos\beta = -\dfrac{11\,753}{15\,625}$

386. a)
$$\left.\begin{array}{l} \text{arc tg}\,\dfrac{1}{2} = \beta \Rightarrow \text{tg}\,\beta = \dfrac{1}{2} \text{ e } -\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2} \Rightarrow 0 < \beta < \dfrac{\pi}{2} \\ \text{arc tg}\,\dfrac{1}{3} = \alpha \Rightarrow \text{tg}\,\alpha = \dfrac{1}{3} \text{ e } -\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2} \Rightarrow 0 < \alpha < \dfrac{\pi}{2} \end{array}\right\} \Rightarrow$$

$\Rightarrow 0 < \beta + \alpha < \pi$ (A); $\gamma = \dfrac{\pi}{4}$ (B)

$$\text{tg}\,(\beta + \alpha) = \frac{\text{tg}\,\beta + \text{tg}\,\alpha}{1 - \text{tg}\,\beta \cdot \text{tg}\,\alpha} = 1 = \text{tg}\,\gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

b) $\left.\begin{array}{l} \text{arc sen}\,\dfrac{1}{\sqrt{5}} = \beta \Rightarrow \text{sen}\,\beta = \dfrac{1}{\sqrt{5}} \text{ e } -\dfrac{\pi}{2} \leqslant \beta \leqslant \dfrac{\pi}{2} \Rightarrow 0 < \beta < \dfrac{\pi}{2} \\ \text{arc cos}\,\dfrac{3}{\sqrt{10}} = \alpha \Rightarrow \cos\alpha = \dfrac{3}{\sqrt{10}} \text{ e } 0 \leqslant \alpha \leqslant \pi \Rightarrow 0 < \alpha < \dfrac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 < \beta + \alpha < \pi$ (A); $\gamma = \dfrac{\pi}{4}$ (B)

$$\text{sen}^2\,\beta = \frac{\text{tg}^2\,\beta}{1 + \text{tg}^2\,\beta} \Rightarrow \text{tg}\,\beta = \frac{1}{2};\ \cos^2\alpha = \frac{1}{1 + \text{tg}^2\,\alpha} \Rightarrow \text{tg}\,\alpha = \frac{1}{3}$$

$$\text{tg}\,(\beta + \alpha) = \frac{\text{tg}\,\beta + \text{tg}\,\alpha}{1 - \text{tg}\,\beta \cdot \text{tg}\,\alpha} = 1 = \text{tg}\,\gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

c) $\left.\begin{array}{l} \text{arc cos}\,\dfrac{3}{5} = \alpha \Rightarrow \cos\alpha = \dfrac{3}{5} \text{ e } 0 \leqslant \alpha \leqslant \pi \Rightarrow 0 < \alpha < \dfrac{\pi}{2} \\ \text{arc cos}\,\dfrac{12}{13} = \beta \Rightarrow \cos\beta = \dfrac{12}{13} \text{ e } 0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \dfrac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 < \alpha + \beta < \pi$ (A)

$\text{arc cos}\,\dfrac{16}{65} = \gamma \Rightarrow \cos\gamma = \dfrac{16}{65}$ e $0 \leqslant \gamma \leqslant \pi$ (B)

$$\text{sen}\,\alpha = \sqrt{1 - \cos^2\alpha} = \frac{4}{5};\ \text{sen}\,\beta = \sqrt{1 - \cos^2\beta} = \frac{5}{13}$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \text{sen}\,\alpha \cdot \text{sen}\,\beta = \frac{16}{65} = \cos\gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \alpha + \beta = \gamma$.

d) $\left.\begin{array}{l} \text{arc sen}\,\dfrac{24}{25} = \alpha \Rightarrow \text{sen}\,\alpha = \dfrac{24}{25} \text{ e } -\dfrac{\pi}{2} \leqslant \alpha \leqslant \dfrac{\pi}{2} \Rightarrow 0 < \alpha < \dfrac{\pi}{2} \\ \text{arc sen}\,\dfrac{3}{5} = \beta \Rightarrow \text{sen}\,\beta = \dfrac{3}{5} \text{ e } -\dfrac{\pi}{2} \leqslant \beta \leqslant \dfrac{\pi}{2} \Rightarrow 0 < \beta < \dfrac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 < \alpha + \beta < \pi$ (A)

$\text{arc tg}\,\dfrac{3}{4} = \gamma \Rightarrow \text{tg}\,\gamma = \dfrac{3}{4}$ e $-\dfrac{\pi}{2} < \gamma < \dfrac{\pi}{2}$ (B)

$$\text{sen}^2\,\alpha = \frac{\text{tg}^2\,\alpha}{1 + \text{tg}^2\,\alpha} \Rightarrow \text{tg}\,\alpha = \frac{24}{7};\ \text{sen}^2\,\beta = \frac{\text{tg}^2\,\beta}{1 + \text{tg}^2\,\beta} \Rightarrow \text{tg}\,\beta = \frac{3}{4}$$

$$\text{tg}\,(\alpha - \beta) = \frac{\dfrac{24}{7} - \dfrac{3}{4}}{1 + \dfrac{24}{7} \cdot \dfrac{3}{4}} = \frac{3}{4} = \text{tg}\,\gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \alpha - \beta = \gamma$.

387. a) $\text{arc tg}\,\dfrac{2}{3} = \alpha \Rightarrow \text{tg}\,\alpha = \dfrac{2}{3}$ e $-\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2} \Rightarrow 0 < \alpha < \dfrac{\pi}{4} \Rightarrow$

$\Rightarrow 0 < 2\alpha < \dfrac{\pi}{2}$

$\text{arc cos}\,\dfrac{12}{13} = \beta \Rightarrow \cos\beta = \dfrac{12}{13}$ e $0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \dfrac{\pi}{2} \Rightarrow$

$\Rightarrow 0 < 2\alpha + \beta < \pi$ (A)

$$\text{tg}\,2\alpha = \dfrac{2\cdot\dfrac{2}{3}}{1-\left(\dfrac{2}{3}\right)^2} = \dfrac{12}{5};\ \cos 2\alpha = \sqrt{\dfrac{1}{1+\left(\dfrac{12}{5}\right)^2}} = \dfrac{5}{13};$$

$$\text{sen}\,2\alpha = \sqrt{1-\left(\dfrac{5}{13}\right)^2} = \dfrac{12}{13}$$

$$\text{sen}\,\beta = \sqrt{1-\left(\dfrac{12}{13}\right)^2} = \dfrac{5}{13};\ \gamma = \dfrac{\pi}{2}\ \text{(B)}$$

$$\cos(2\alpha+\beta) = \dfrac{5}{13}\cdot\dfrac{12}{13} - \dfrac{12}{13}\cdot\dfrac{5}{13} = 0 = \cos\gamma\ \text{(C)}$$

De (A), (B) e (C) $\Rightarrow 2\alpha + \beta = \gamma$.

b) $\left.\begin{array}{l} \text{arc sen}\,\dfrac{1}{4} = \alpha \Rightarrow \text{sen}\,\alpha = \dfrac{1}{4}\ \text{e}\ -\dfrac{\pi}{2} \leqslant \alpha \leqslant \dfrac{\pi}{2} \Rightarrow 0 < \alpha < \dfrac{\pi}{6} \Rightarrow \\ \Rightarrow 0 < 3\alpha < \dfrac{\pi}{2} \\ \text{arc cos}\,\dfrac{11}{16} = \beta \Rightarrow \cos\beta = \dfrac{11}{16}\ \text{e}\ 0 \leqslant \beta \leqslant \pi \Rightarrow 0 < \beta < \dfrac{\pi}{2} \end{array}\right\} \Rightarrow$

$\Rightarrow 0 < 3\alpha + \beta < \pi$ (A); $\gamma = \dfrac{\pi}{2}$ (B)

$$\cos\alpha = \sqrt{1-\dfrac{1}{16}} = \dfrac{\sqrt{15}}{4},\ \text{sen}\,\beta = \sqrt{1-\dfrac{121}{256}} = \dfrac{3\sqrt{15}}{16};$$

$$\cos 3\alpha = 4\cdot\left(\dfrac{\sqrt{15}}{4}\right)^3 - \dfrac{3\sqrt{15}}{4} = \dfrac{3\sqrt{15}}{16}$$

$$\text{sen}\,3\alpha = 3\cdot\dfrac{1}{4} - 4\left(\dfrac{1}{4}\right)^3 = \dfrac{11}{16},\ \cos(3\alpha+\beta) =$$

$$= \dfrac{3\sqrt{15}}{16}\cdot\dfrac{11}{16} - \dfrac{11}{16}\cdot\dfrac{3\sqrt{15}}{16} = 0 = \cos\gamma\ \text{(C)}$$

De (A), (B) e (C) $\Rightarrow 3\alpha + \beta = \gamma$.

388. $\text{arc tg}\left(\dfrac{1+e^x}{2}\right) = \alpha \Rightarrow \text{tg}\,\alpha = \dfrac{1+e^x}{2}$ e $-\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2}$

$\text{arc tg}\left(\dfrac{1-e^x}{2}\right) = \beta \Rightarrow \text{tg}\,\beta = \dfrac{1-e^x}{2}$ e $-\dfrac{\pi}{2} < \beta < \dfrac{\pi}{2}$

$$\alpha + \beta = \frac{\pi}{4} \Rightarrow \operatorname{tg}(\alpha + \beta) = \frac{\left(\frac{1 + e^x}{2}\right) + \left(\frac{1 - e^x}{2}\right)}{1 - \left(\frac{1 + e^x}{2}\right)\left(\frac{1 - e^x}{2}\right)} =$$

$$= \frac{1}{1 - \frac{1 - e^{2x}}{4}} = \frac{4}{3 + e^{2x}} \Rightarrow \frac{4}{3 + e^{2x}} = 1 \Rightarrow e^{2x} = 1 \Rightarrow x = 0,\ S = \{0\}$$

389. $\alpha = \operatorname{arc\,tg}(7x - 1) \Rightarrow \operatorname{tg}\alpha = 7x - 1$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ (A)

$\beta = \operatorname{arc\,sec}(2x + 1) \Rightarrow \sec\beta = 2x + 1$ e $0 \leqslant \beta \leqslant \pi$ (B)

$\alpha = \beta \Rightarrow \operatorname{tg}\alpha = \operatorname{tg}\beta \Rightarrow \operatorname{tg}^2\alpha = \operatorname{tg}^2\beta \Rightarrow \operatorname{tg}^2\alpha = \sec^2\beta - 1 \Rightarrow$

$\Rightarrow (7x - 1)^2 = (2x + 1)^2 - 1 \Rightarrow 45x^2 - 18x + 1 = 0 \Rightarrow$

$\Rightarrow x = \frac{1}{3}$ ou $x = \frac{1}{15}$ (não satisfaz $\operatorname{tg}\alpha = -\operatorname{tg}\beta$) $\therefore x = \frac{1}{3}$

390. $\alpha = \operatorname{arc\,sen}\left(\frac{4}{5}\right) \Rightarrow \operatorname{sen}\alpha = \frac{4}{5}$ e $\frac{\pi}{2} < \alpha < \pi$

$\beta = \operatorname{arc\,tg}\left(-\frac{4}{3}\right) \Rightarrow \operatorname{tg}\beta = -\frac{4}{3}$ e $\frac{3\pi}{2} < \beta \leqslant 2\pi$

$\cos\alpha = -\sqrt{1 - \frac{16}{25}} = -\frac{3}{5}$; $\operatorname{sen}\beta = -\sqrt{\frac{\frac{16}{9}}{1 + \frac{16}{9}}} = -\frac{4}{5}$;

$\cos\beta = \sqrt{1 - \left(-\frac{4}{5}\right)^2} = \frac{3}{5}$

$25 \cdot \cos(\alpha + \beta) = 25 \cdot \left[-\frac{3}{5} \cdot \frac{3}{5} - \frac{4}{5}\left(-\frac{4}{5}\right)\right] = 7$

Apêndice A — Resolução de equações e inequações em intervalos determinados

393. $\operatorname{sen} 3x = \operatorname{sen}\frac{\pi}{6} \Rightarrow \begin{cases} 3x = \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{\pi}{18} + 2k\frac{\pi}{3} \\ \text{ou} \\ 3x = \pi - \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{5\pi}{18} + 2k\frac{\pi}{3} \end{cases}$

$k = 0 \Rightarrow x = \frac{\pi}{18}$ ou $x = \frac{5\pi}{18}$; $k = 1 \Rightarrow x = \frac{13\pi}{18}$ ou $x = \frac{17\pi}{18}$

$S = \left\{\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}\right\}$

395. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = \pi - 2x + 2k\pi \Rightarrow x = \dfrac{\pi}{5} + \dfrac{2k\pi}{5}$

$S = \left\{0, \dfrac{\pi}{5}, \dfrac{3\pi}{5}, \pi\right\}$

397. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = -2x + 2k\pi \Rightarrow x = \dfrac{2k\pi}{5}$

$S = \left\{0, \dfrac{2\pi}{5}, \dfrac{4\pi}{5}\right\}$

398. $\text{sen}\, x \cdot (4\, \text{sen}^2\, x - 1) = 0$. Então: $\text{sen}\, x = 0 \Rightarrow x = k\pi$ ou $\text{sen}\, x = \dfrac{1}{2} \Rightarrow$

$\Rightarrow x = \dfrac{\pi}{6} + 2k\pi$ ou $x = \dfrac{5\pi}{6} + 2k\pi$ ou $\text{sen}\, x = -\dfrac{1}{2} \Rightarrow$

$\Rightarrow x = \dfrac{7\pi}{6} + 2k\pi$ ou $x = -\dfrac{\pi}{6} + 2k\pi \Rightarrow$

$\Rightarrow S = \left\{0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \pi, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, 2\pi\right\}$

400. $\text{tg}\, x + \dfrac{1}{\text{tg}\, x} = 2 \Rightarrow \text{tg}^2\, x - 2\, \text{tg}\, x + 1 = 0 \Rightarrow$

$\Rightarrow \text{tg}\, x = 1 \Rightarrow x = \dfrac{\pi}{4} + k\pi \Rightarrow S = \left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$

401. $\sqrt{\text{sen}^2\, x} = \sqrt{\cos^2 x} \Rightarrow \text{sen}^2\, x = \cos^2 x \Rightarrow \cos^2 x - \text{sen}^2\, x = 0 \Rightarrow$

$\Rightarrow \cos 2x = 0 \Rightarrow 2x = \dfrac{\pi}{2} + k\pi \Rightarrow x = \dfrac{\pi}{4} + \dfrac{k\pi}{2} \Rightarrow S = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}$

402. $\text{sen}\, x + \text{sen}\, y = 2\, \text{sen}\left(\dfrac{x+y}{2}\right) \cdot \cos\left(\dfrac{x-y}{2}\right) = 1 \Rightarrow \cos\left(\dfrac{\frac{\pi}{3} - 2y}{2}\right) = 1 \Rightarrow$

$\Rightarrow \cos\left(\dfrac{\pi}{6} - y\right) = \cos 0 \Rightarrow \dfrac{\pi}{6} - y = 2k\pi \Rightarrow y = \dfrac{\pi}{6} - 2k\pi$

$x + y = \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{6} + 2k\pi$

403. a) $\cos 2x = \cos \dfrac{\pi}{6} \Rightarrow \begin{cases} 2x = \dfrac{\pi}{6} + 2k\pi \Rightarrow x = \dfrac{\pi}{12} + k\pi \\ \text{ou} \qquad\qquad\qquad\qquad \text{ou} \\ 2x = -\dfrac{\pi}{6} + 2k\pi \Rightarrow x = -\dfrac{\pi}{12} = k\pi \end{cases}$

$S = \left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}$

b) $2x = x + 2k\pi \Rightarrow x = 2k\pi$ ou $2x = -x + 2k\pi \Rightarrow x = \dfrac{2k\pi}{3}$

$S = \left\{0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 2\pi\right\}$

c) $\cos\left(x + \dfrac{\pi}{6}\right) = \cos \dfrac{\pi}{2} \Rightarrow x = \dfrac{\pi}{3} + k\pi \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{4\pi}{3}\right\}$

404. a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \dfrac{k\pi}{2}$

$S = \left\{0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, 2\pi\right\}$

b) $5x = x + \dfrac{\pi}{3} + 2k\pi \Rightarrow x = \dfrac{\pi}{12} + \dfrac{k\pi}{2}$ ou $5x = -x - \dfrac{\pi}{3} + 2k\pi \Rightarrow$

$\Rightarrow x = -\dfrac{\pi}{18} + \dfrac{k\pi}{3}$

$S = \left\{\dfrac{\pi}{12}, \dfrac{7\pi}{12}, \dfrac{13\pi}{12}, \dfrac{19\pi}{12}, \dfrac{5\pi}{18}, \dfrac{11\pi}{18}, \dfrac{17\pi}{18}, \dfrac{23\pi}{18}, \dfrac{29\pi}{18}, \dfrac{35\pi}{18}\right\}$

405. $\cos^2 x - \dfrac{2}{\cos^2 x} = 1 \Rightarrow \cos^4 x - \cos^2 x - 2 = 0 \Rightarrow \cos^2 x = 2$ ou

$\cos^2 x = -1$, $S = \varnothing$

406. O 1º membro é a soma dos 10 termos da P.G., com $a_1 = 1$ e $q = \cos x$; então sua soma é $\dfrac{1 \cdot \cos^{10} x - 1}{\cos x - 1} = 0$, e daí $\cos x = -1$; a equação tem uma única solução.

407. a) $\operatorname{tg} 2x = \operatorname{tg} \dfrac{\pi}{3} \Rightarrow x = \dfrac{\pi}{6} + \dfrac{k\pi}{2} \Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}\right\}$

b) $2x = x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$

c) $\operatorname{tg} 3x = \operatorname{tg} \dfrac{\pi}{4} \Rightarrow 3x = \dfrac{\pi}{4} + k\pi \Rightarrow x = \dfrac{\pi}{12} + \dfrac{k\pi}{3} \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}, \dfrac{13\pi}{12}, \dfrac{17\pi}{12}, \dfrac{21\pi}{12}\right\}$

d) $\operatorname{tg} 3x = \operatorname{tg} 2x \Rightarrow 3x = 2x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$

e) $2x = x + \dfrac{\pi}{4} + k\pi \Rightarrow x = \dfrac{\pi}{4} + k\pi$, mas $x + \dfrac{\pi}{4} \neq \dfrac{\pi}{2} + k\pi$, então $S = \varnothing$

f) $\operatorname{tg} 4x = \operatorname{tg} \dfrac{\pi}{4} \Rightarrow 4x = \dfrac{\pi}{4} + k\pi \Rightarrow x = \dfrac{\pi}{16} + \dfrac{k\pi}{4} \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{16}, \dfrac{5\pi}{16}, \dfrac{9\pi}{16}, \dfrac{13\pi}{16}, \dfrac{17\pi}{16}, \dfrac{21\pi}{16}, \dfrac{25\pi}{16}, \dfrac{29\pi}{16}\right\}$

g) $2x = x + \dfrac{\pi}{4} + k\pi \Rightarrow x = \dfrac{\pi}{4} + k\pi \Rightarrow S = \left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$

h) $\operatorname{tg} 2x = \operatorname{tg} \dfrac{\pi}{3} \Rightarrow 2x = \dfrac{\pi}{3} + k\pi \Rightarrow x = \dfrac{\pi}{6} + \dfrac{k\pi}{2}$

ou

$\operatorname{tg} 2x = \operatorname{tg} \dfrac{2\pi}{3} \Rightarrow 2x = \dfrac{2\pi}{3} + k\pi \Rightarrow x = \dfrac{\pi}{3} + \dfrac{k\pi}{2} \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{2\pi}{3}, \dfrac{7\pi}{6}, \dfrac{5\pi}{3}, \dfrac{\pi}{3}, \dfrac{5\pi}{6}, \dfrac{4\pi}{3}, \dfrac{11\pi}{6}\right\}$

408. a) $1 + tg^2\,x - 2\,tg\,x = 0 \Rightarrow tg\,x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

b) $1 = sen^2 x - sen\,x \cdot cos\,x \Rightarrow sen^2 x + cos^2 x = sen^2 x - sen\,x \cdot cos\,x \Rightarrow$
$\Rightarrow cos\,x\,(sen\,x + cos\,x) = 0$

Então:

$cos\,x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$ ou $sen\,x + cos\,x = 0 \Rightarrow sen\,x = -cos\,x \Rightarrow$

$\Rightarrow tg\,x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi \Rightarrow S = \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

c) $sen\,2x \cdot cos\left(x + \frac{\pi}{4}\right) - cos\,2x \cdot sen\left(x + \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow sen\left[2x - \left(x + \frac{\pi}{4}\right)\right] = 0 \Rightarrow sen\left(x - \frac{\pi}{4}\right) = 0 \Rightarrow$

$\Rightarrow x - \frac{\pi}{4} = k\pi \Rightarrow S = \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

d) $1 + sen\,2x - tg\,x - tg\,x\,sen\,2x = 1 + tg\,x \Rightarrow sen\,2x(1 - tg\,x) = 2\,tg\,x \Rightarrow$

$\Rightarrow \frac{2\,tg\,x}{1 + tg^2\,x} = \frac{2\,tg\,x}{1 - tg\,x} \Rightarrow tg\,x(tg\,x + 1) = 0$. Então: $tg\,x = 0 \Rightarrow$

$\Rightarrow x = k\pi$ ou

$tg\,x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$ e daí $S = \left\{0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi\right\}$

e) $sec\,x(3\,sec\,x - 2) = 0 \Rightarrow sec\,x = 0$ ou $sec\,x = \frac{2}{3}$ e daí $S = \varnothing$

f) $2\,sen^2 x = \frac{sen\,x}{cos\,x} \cdot cos\,x \Rightarrow sen\,x(2\,sen\,x - 1) = 0$. Então: $sen\,x = 0 \Rightarrow$

$\Rightarrow x = k\pi$ ou $sen\,x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} = 2k\pi \Rightarrow$

$\Rightarrow S = \left\{0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi\right\}$

409. $\frac{\pi}{2}p = \frac{\pi}{4} + \frac{k\pi}{2} \Rightarrow p = \frac{1}{2} + k$

$tg\,\frac{\pi}{2}p = \frac{1}{tg\,\frac{\pi}{2}p} \Rightarrow tg\,\frac{\pi}{2}p = \pm 1$

410. $sen\,x = tg\,x \Rightarrow cos\,x = 1 \Rightarrow x = 2k\pi \Rightarrow \nexists x \mid 0 < x < \pi \Rightarrow$ nenhum ponto

411. $1 + tg^2\,x - tg\,x = 1 \Rightarrow tg\,x(tg\,x - 1) = 0 \Rightarrow tg\,x = 0$ ou $tg\,x = 1 \Rightarrow$

$\Rightarrow x = k\pi$ ou $x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi\right\}$

412. $6x = 2x + k\pi \Rightarrow x = \dfrac{k\pi}{4} \Rightarrow x \in \left\{0, \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}, \pi\right\}$

$x = \dfrac{\pi}{4} \Rightarrow \nexists \text{ tg } 2x;\ x = \dfrac{3\pi}{4} \Rightarrow \nexists \text{ tg } 2x \Rightarrow S = \left\{0, \dfrac{\pi}{2}, \pi\right\}$

413. a) $(\text{sen}^2 x + \cos^2 x)^2 - 2\,\text{sen}^2 x \cos^2 x = \dfrac{5}{8} \Rightarrow \text{sen } 2x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{6} + k\pi \text{ ou } x = \dfrac{\pi}{3} + k\pi \text{ ou } x = \dfrac{2\pi}{3} + k\pi \text{ ou } x = \dfrac{5\pi}{6} + k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}, \dfrac{11\pi}{6}\right\}$

b) $(\text{sen}^2 x + \cos^2 x)(\text{sen}^4 x - \text{sen}^2 x \cos^2 x + \cos^4 x) = \dfrac{5}{8} \Rightarrow$

$\Rightarrow \text{sen } 2x = \pm\dfrac{\sqrt{2}}{2} \Rightarrow \left(x = \dfrac{\pi}{8} + k\pi \text{ ou } x = \dfrac{3\pi}{8} + k\pi \text{ ou } x = \dfrac{5\pi}{8} + k\pi\right.$

$\left.\text{ou } \dfrac{7\pi}{8} + k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}\right\}$

c) $\text{sen}^2 2x = 1 \Rightarrow \text{sen } 2x = \pm 1 \Rightarrow \left(x = \dfrac{\pi}{4} + k\pi \text{ ou } x = \dfrac{3\pi}{4} + k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$

d) $\text{sen}^2 x = \dfrac{3}{4} \Rightarrow \text{sen } x = \pm\dfrac{\sqrt{3}}{2} \Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = \dfrac{4\pi}{3} + 2k\pi \text{ ou}\right.$

$\left.x = \dfrac{2\pi}{3} + 2k\pi \text{ ou } x = \dfrac{5\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}$

e) $(\text{sen } x + \cos x)(\text{sen}^2 x - \text{sen } x \cdot \cos x + \cos^2 x) = 1 \Rightarrow$

$\Rightarrow (\text{sen } x + \cos x)(1 - \text{sen } x \cdot \cos x) = 1$; fazendo $\text{sen } x + \cos x = y$

e $\text{sen } x \cdot \cos x = \dfrac{y^2 - 1}{2}$, vem $y \cdot \left(1 - \dfrac{y^2 - 1}{2}\right) = 1 \Rightarrow$

$\Rightarrow y = 1$ ou $y = -2$ (não serve, pois $-\sqrt{2} \leqslant y \leqslant \sqrt{2}$)

$\text{sen } x + \cos x = 1 \Rightarrow \sqrt{2} \cos\left(x - \dfrac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{2} + 2k\pi \text{ ou } x = 2k\pi\right) \Rightarrow S = \left\{0, \dfrac{\pi}{2}, 2\pi\right\}$

414. $2x = x + 2k\pi \Rightarrow x = 2k\pi \text{ ou } 2x = \pi - x + 2k\pi \Rightarrow x = \dfrac{\pi}{3} + \dfrac{2k\pi}{3} \Rightarrow$

$\Rightarrow S = \left\{0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}\right\} \Rightarrow$ quatro soluções

415. $\dfrac{3\,\text{sen}^2\,x}{\cos^2 x} + 5 = \dfrac{7}{\cos x} \Rightarrow 3\,(1 - \cos^2 x) + 5\cos^2 x - 7\cos x = 0 \Rightarrow$

$\Rightarrow 2\cos^2 x - 7\cos x + 3 = 0 \Rightarrow \left(\cos x = 3 \text{ impossível ou } \cos x = \dfrac{1}{2}\right) \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = -\dfrac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{-\dfrac{\pi}{3}, +\dfrac{\pi}{3}\right\}$

416. $\text{sen}\,\pi x = -\cos \pi x \Rightarrow \text{sen}\,\pi x = \text{sen}\left(\dfrac{3\pi}{2} - \pi x\right) \Rightarrow x = \dfrac{3}{4} + k \Rightarrow$

$\Rightarrow S = \left\{\dfrac{3}{4}, \dfrac{7}{4}\right\}$

417. $1 + \text{tg}^2\,x = \text{tg}\,x + 1 \Rightarrow \text{tg}\,x\,(\text{tg}\,x - 1) = 0 \Rightarrow (\text{tg}\,x = 0 \text{ ou } \text{tg}\,x = 1) \Rightarrow$

$\Rightarrow \left(x = k\pi \text{ ou } x = \dfrac{\pi}{4} + k\pi\right) \Rightarrow S = \left\{0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}, 2\pi\right\}$

418. a) $2\,(1 - \cos^2 x) - 3\cos x - 3 = 0 \Rightarrow 2\cos^2 x + 3\cos x + 1 = 0 \Rightarrow$

$\Rightarrow \left(\cos x = -\dfrac{1}{2} \text{ ou } \cos x = -1\right) \Rightarrow$

$\Rightarrow \left(x = \dfrac{2\pi}{3} + 2k\pi \text{ ou } x = \dfrac{4\pi}{3} + 2k\pi \text{ ou } x = \pi + 2k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{\dfrac{2\pi}{3}, \pi, \dfrac{4\pi}{3}\right\}$

b) $\cos^2 x - 2\cos x + 1 = 0 \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi \Rightarrow S = \{0, 2\pi\}$

c) $2\cos^2 x + 5\cos x - 3 = 0 \Rightarrow \cos x = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = -\dfrac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}$

d) $4\cos^2 x - 8\cos x + 3 = 0 \Rightarrow \cos x = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = -\dfrac{\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}$

419. $\cos x = \pm\dfrac{1}{2} \Rightarrow \left(x = \dfrac{\pi}{3} + 2k\pi \text{ ou } x = \dfrac{5\pi}{3} + 2k\pi \text{ ou } x = \dfrac{2\pi}{3} + 2k\pi\right.$

$\left.\text{ou } x = \dfrac{4\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\} \Rightarrow$

$\Rightarrow \text{soma} = \dfrac{\pi + 2\pi + 4\pi + 5\pi}{3} = 4\pi$

420. $2\cos^2 x + 3\,(1 - \cos^2 x) - 5 - 3\cos x = 0 \Rightarrow \cos^2 x + 3\cos x + 2 = 0 \Rightarrow$

$\Rightarrow \cos x = -1 \Rightarrow x = \pi + 2k\pi \Rightarrow S = \{\pi\} \Rightarrow$ uma solução

421. a) $\operatorname{sen}(x+y)+\operatorname{sen}(x-y)=\operatorname{sen}\frac{5\pi}{2}+\operatorname{sen}\left(\frac{3\pi}{2}\right)=1-1=0\neq 2$

b) $\left.\begin{array}{l}2\operatorname{sen}x\cos y=2\\ \operatorname{sen}x+\cos y=2\end{array}\right\}\Rightarrow \left.\begin{array}{ll}\operatorname{sen}x\cos y=1 & \text{(A)}\\ \operatorname{sen}x+\cos y=2 & \text{(B)}\end{array}\right\}\text{(A) em (B)}\Rightarrow$

$\Rightarrow \operatorname{sen}^2 x-2\operatorname{sen}x+1=0\Rightarrow \operatorname{sen}x=1\ \text{(C)}\Rightarrow x=\frac{\pi}{2}+2k\pi$

(C) em (A) $\Rightarrow \cos y=1\Rightarrow y=2k\pi$

$k=\{0,1\}\Rightarrow S=\left\{\left(\frac{\pi}{2},0\right)\right\}$ ou $\left\{\left(\frac{\pi}{2},2\pi\right)\right\}$

422. $\begin{cases}\operatorname{sen}a+\cos b=1\ \text{(A)}\\ \operatorname{sen}a+\operatorname{sen}b=1\ \text{(B)}\end{cases}$ (A) em (B) $\Rightarrow \operatorname{sen}b-\cos b=0\Rightarrow$

$\Rightarrow \operatorname{sen}\left(b-\frac{\pi}{4}\right)=0\Rightarrow b=\frac{\pi}{4}+k\pi\Rightarrow b=\frac{\pi}{4}\ \text{(C)}$

(C) em (A) $\Rightarrow \operatorname{sen}a=\frac{2-\sqrt{2}}{2}\Rightarrow a=\operatorname{arc\,sen}\frac{2-\sqrt{2}}{2}$

$S=\left\{\left(\operatorname{arc\,sen}\frac{2-\sqrt{2}}{2},\frac{\pi}{4}\right)\right\}$

423. 1º caso: $\operatorname{sen}x\geqslant 0$

$|\operatorname{sen}x|=\operatorname{sen}x\Rightarrow 2\operatorname{sen}^2x+\operatorname{sen}x-1=0\Rightarrow \operatorname{sen}x=\frac{1}{2}\Rightarrow$

$\Rightarrow x=\frac{\pi}{6}+2k\pi$ ou $x=\frac{5\pi}{6}+2k\pi$

2º caso: $\operatorname{sen}x<0$

$|\operatorname{sen}x|=\operatorname{sen}x\Rightarrow 2\operatorname{sen}^2x-\operatorname{sen}x-1=0\Rightarrow \operatorname{sen}x=-\frac{1}{2}\Rightarrow$

$\Rightarrow x=\frac{7\pi}{6}+2k\pi$ ou $x=\frac{11\pi}{6}+2k\pi$

Então: $S=\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$.

424. $\cos x=\pm\frac{1}{2}\Rightarrow\left(x=\frac{\pi}{3}+2k\pi\text{ ou }x=\frac{5\pi}{3}+2k\pi\text{ ou }x=\frac{2\pi}{3}+2k\pi\right.$

$\left.\text{ou }x=\frac{4\pi}{3}+2k\pi\right)\Rightarrow S=\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}\Rightarrow$ a soma é π

425. $\log 2\operatorname{sen}^2x=0\Rightarrow 2\operatorname{sen}^2x=1\Rightarrow \operatorname{sen}x=\pm\frac{\sqrt{2}}{2}\Rightarrow$

$\Rightarrow\left(x=\frac{\pi}{4}+2k\pi\text{ ou }x=\frac{3\pi}{4}+2k\pi\text{ ou }x=\frac{5\pi}{4}+2k\pi\text{ ou }x=\frac{7\pi}{4}+2k\pi\right)\Rightarrow$

$\Rightarrow S=\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$

426. $\text{sen } x = y \Rightarrow 2y^2 - 5y + 2 = 0 \Rightarrow$ $y = 2$ não serve ou $y = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \text{sen } x = \dfrac{1}{2} \Rightarrow \left(x = \dfrac{\pi}{6} + 2k\pi \text{ ou } x = \dfrac{5\pi}{6} + 2k\pi\right) \Rightarrow$

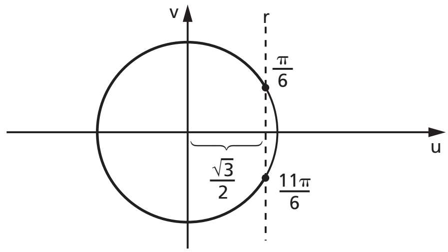
$\Rightarrow S = \left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\} \Rightarrow$ a soma é π

428. $2x = t \Rightarrow \cos t \leqslant \dfrac{\sqrt{3}}{2}$

$\dfrac{\pi}{6} + 2k\pi \leqslant t \leqslant \dfrac{11\pi}{6} + 2k\pi$

$\dfrac{\pi}{12} + k\pi \leqslant x \leqslant \dfrac{11\pi}{12} + k\pi$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{12} \leqslant x \leqslant \dfrac{11\pi}{12} \text{ ou } \dfrac{13\pi}{12} \leqslant x \leqslant \dfrac{23\pi}{12}\right\}$



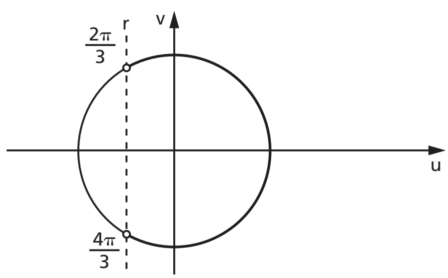
429. $4x = t \Rightarrow \cos t > -\dfrac{1}{2}$

$2k\pi \leqslant t < \dfrac{2\pi}{3} + 2k\pi \text{ ou } \dfrac{4\pi}{3} + 2k\pi < t \leqslant 2\pi + 2k\pi$

$\dfrac{k\pi}{2} \leqslant x < \dfrac{\pi}{6} + \dfrac{k\pi}{2} \text{ ou } \dfrac{\pi}{3} + \dfrac{k\pi}{2} < x \leqslant \dfrac{\pi}{2} + \dfrac{k\pi}{2}$

$k = \{0, 1, 2\} \Rightarrow S = \Big\{x \in \mathbb{R} \mid 0 \leqslant x < \dfrac{\pi}{6} \text{ ou } \dfrac{\pi}{3} < x < \dfrac{2\pi}{3} \text{ ou}$

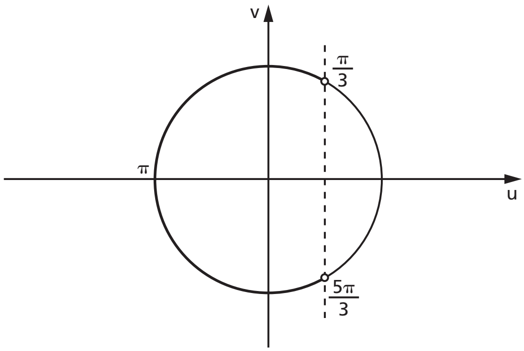
$\dfrac{5\pi}{6} < x < \dfrac{7\pi}{6} \text{ ou } \dfrac{4\pi}{3} < x < \dfrac{5\pi}{3} \text{ ou } \dfrac{11\pi}{6} < x \leqslant 2\pi\Big\}$



431. $\cos x = y \Rightarrow \dfrac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \dfrac{1}{2}$ ou $y > 1$

$-1 < \cos x < \dfrac{1}{2}$ ou $\cos x > 1$ impossível

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{3} < x < \pi\right\}$



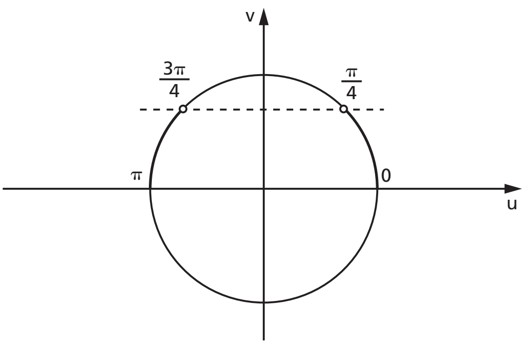
432. $\dfrac{1 - 2\,\text{sen}^2\,x + \text{sen}\,x + 1}{1 - 2\,\text{sen}^2\,x} - 2 \geqslant 0 \Rightarrow$

$\Rightarrow \dfrac{2\,\text{sen}^2\,x + \text{sen}\,x}{1 - 2\,\text{sen}^2\,x} \geqslant 0$, $\text{sen}\,x = y \Rightarrow$

$\Rightarrow \dfrac{2y^2 + y}{1 - 2y^2} \geqslant 0 \Rightarrow -\dfrac{\sqrt{2}}{2} < y \leqslant -\dfrac{1}{2}$ ou $0 \leqslant y < \dfrac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow -\dfrac{\sqrt{2}}{2} < \text{sen}\,x \leqslant -\dfrac{1}{2}$ ou $0 \leqslant \text{sen}\,x < \dfrac{\sqrt{2}}{2} \Rightarrow$

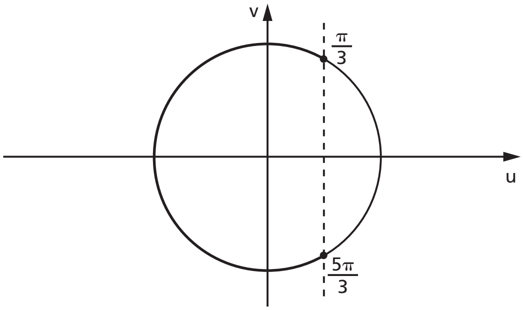
$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \dfrac{\pi}{4} \text{ ou } \dfrac{3\pi}{4} < x \leqslant \pi\right\}$



433. $2^{\cos 2x} \leqslant 2^{\frac{1}{2}} \Rightarrow \cos 2x \leqslant \dfrac{1}{2}$, $2x = t \Rightarrow \cos t \leqslant \dfrac{1}{2} \Rightarrow$

$\Rightarrow \dfrac{\pi}{3} + 2k\pi \leqslant t \leqslant \dfrac{5\pi}{3} + 2k\pi \Rightarrow \dfrac{\pi}{6} + k\pi \leqslant x \leqslant \dfrac{5\pi}{6} + k\pi \Rightarrow$

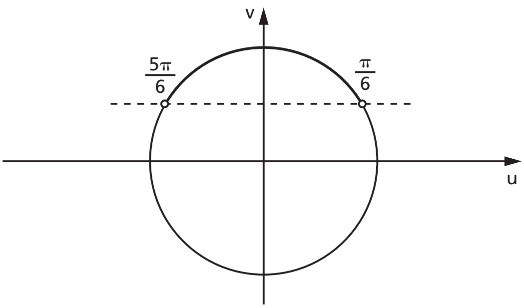
$\Rightarrow S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}\right\}$



435. $2x = t \Rightarrow \text{sen } t > \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} + 2k\pi < t < \dfrac{5\pi}{6} + 2k\pi \Rightarrow$

$\Rightarrow \dfrac{\pi}{12} + k\pi < x < \dfrac{5\pi}{12} + k\pi$

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{12} < x < \dfrac{5\pi}{12} \text{ ou } \dfrac{13\pi}{12} < x < \dfrac{17\pi}{12}\right\}$



436. $3x = t \Rightarrow \text{sen } t \leqslant \dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \left(2k\pi \leqslant t \leqslant \dfrac{\pi}{3} + 2k\pi \text{ ou } \dfrac{2\pi}{3} + 2k\pi \leqslant t \leqslant 2\pi + 2k\pi\right) \Rightarrow$

$\Rightarrow \left(\dfrac{2k\pi}{3} \leqslant x \leqslant \dfrac{\pi}{9} + \dfrac{2k\pi}{3} \text{ ou } \dfrac{2\pi}{9} + \dfrac{2k\pi}{3} \leqslant x \leqslant \dfrac{2\pi}{3} + \dfrac{2k\pi}{3}\right) \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \dfrac{\pi}{9} \text{ ou } \dfrac{2\pi}{9} \leqslant x \leqslant \dfrac{7\pi}{9} \text{ ou } \dfrac{8\pi}{9} \leqslant x \leqslant \dfrac{13\pi}{9} \text{ ou } \dfrac{14\pi}{9} \leqslant x \leqslant 2\pi\right\}$

437. $\frac{1}{4} \leqslant \frac{1}{2}$ sen $2x < \frac{1}{2} \Rightarrow \frac{1}{2} \leqslant$ sen $t < 1, t = 2x,$

$\frac{\pi}{6} + 2k\pi \leqslant t \leqslant \frac{5\pi}{6} + 2k\pi, t \neq \frac{\pi}{2} + 2k\pi \Rightarrow \frac{\pi}{12} + k\pi \leqslant x \leqslant \frac{5\pi}{12} + k\pi,$

$x \neq \frac{\pi}{4} + k\pi \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid \frac{\pi}{12} \leqslant x \leqslant \frac{5\pi}{12}, x \neq \frac{\pi}{4} \text{ ou } \frac{13\pi}{12} \leqslant x \leqslant \frac{17\pi}{12}, x \neq \frac{5\pi}{4}\right\}$

438. $\frac{4\,\text{sen}^2 x - 1}{\cos x} \geqslant 0 \Rightarrow \frac{4(1 - \cos^2 x) - 1}{\cos x} \geqslant 0 \Rightarrow \frac{3 - 4\cos^2 x}{\cos x} \geqslant 0 \Rightarrow$

$\Rightarrow \left(\cos x \leqslant -\frac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leqslant \frac{\sqrt{3}}{2}\right) \Rightarrow$

$\Rightarrow \left(\frac{5\pi}{6} \leqslant x \leqslant \frac{7\pi}{6} \text{ ou } \frac{\pi}{6} \leqslant x < \frac{\pi}{2} \text{ ou } \frac{3\pi}{2} < x \leqslant \frac{11\pi}{6}\right)$

439. $-1 \leqslant \text{sen}\,2x \leqslant +1 \Rightarrow -1 - 2 \leqslant \text{sen}\,2x - 2 \leqslant 1 - 2 \Rightarrow \text{sen}\,2x - 2 < 0\,(A)$

$\frac{\text{sen}\,2x - 2}{\cos 2x + 3\cos x - 1} \geqslant 0 \overset{(A)}{\Rightarrow} \cos 2x + 3\cos x - 1 < 0 \Rightarrow$

$\Rightarrow 2\cos^2 x + 3\cos x - 2 < 0 \Rightarrow -2 < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{5\pi}{3}$

440. a) $\Delta \geqslant 0 \Rightarrow [-(4\cos\alpha)]^2 - 4(2\cos^2\alpha)(4\cos^2\alpha - 1) \geqslant 0 \Rightarrow$

$\Rightarrow -32\cos^4\alpha + 24\cos^2\alpha \geqslant 0$ fazendo $\cos\alpha = t \Rightarrow$

$\Rightarrow -32t^4 + 24t^2 \geqslant 0 \Rightarrow$

$\Rightarrow -8t^2(4t^2 - 3) \geqslant 0 \Rightarrow -\frac{\sqrt{3}}{2} \leqslant t \leqslant \frac{\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2} \leqslant \cos\alpha \leqslant \frac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow S = \left\{\alpha \in \mathbb{R} \mid \frac{\pi}{6} \leqslant \alpha \leqslant \frac{5\pi}{6}\right\}$

b) $-\frac{b}{a} < 0 \Rightarrow \frac{4\cos\alpha}{2\cos^2\alpha} < 0 \Rightarrow \frac{2}{\cos\alpha} < 0 \Rightarrow \cos\alpha < 0 \Rightarrow \frac{\pi}{2} < \alpha \leqslant \pi\,(A)$

$\frac{c}{a} > 0 \Rightarrow \frac{4\cos^2\alpha - 1}{2\cos^2\alpha} > 0 \Rightarrow 4\cos^2\alpha - 1 > 0$

Fazendo $\cos\alpha = y \Rightarrow 4y^2 - 1 > 0 \Rightarrow \left(y < -\frac{1}{2} \text{ ou } y > \frac{1}{2}\right) \Rightarrow$

$\Rightarrow \left(\cos\alpha < -\frac{1}{2} \text{ ou } \cos\alpha > \frac{1}{2}\right) \Rightarrow \left(\frac{2\pi}{3} < \alpha \leqslant \pi \text{ ou } 0 \leqslant \alpha < \frac{\pi}{3}\right)(B)$

Soluções reais $\Rightarrow \frac{\pi}{6} \leqslant \alpha \leqslant \frac{5\pi}{6}$ (C),

$A \cap B \cap C = \left\{\alpha \in \mathbb{R} \mid \frac{2\pi}{3} < \alpha \leqslant \frac{5\pi}{6}\right\}$

441. $(\cos x > 0 \text{ e } 2 \cdot \cos x - 1 > 0 \text{ e } 1 + \cos x > 0) \Rightarrow \cos x > \frac{1}{2}$ (A)

$\log_{\cos x} (2 \cos x - 1)(1 + \cos x) > \log_{\cos x} \cos x \Rightarrow$

$\Rightarrow 2 \cos^2 x + \cos x - \cos x - 1 < 0 \Rightarrow$

$\Rightarrow 2 \cdot \cos^2 x - 1 < 0 \Rightarrow -\frac{\sqrt{2}}{2} < \cos x < \frac{\sqrt{2}}{2}$ (B).

De (A) e (B) vem $\frac{1}{2} < \cos x < \frac{\sqrt{2}}{2}$, então:

$S = \left\{x \in \mathbb{R} \mid \frac{\pi}{4} < x < \frac{\pi}{3} \text{ ou } \frac{5\pi}{3} < x < \frac{7\pi}{4}\right\}$

442. Uma condição necessária é $\operatorname{sen} x > 0$. Então: $\left(\sqrt{1 - \cos x}\right)^2 < \operatorname{sen}^2 x \Rightarrow$
$\Rightarrow 1 - \cos x < 1 - \cos^2 x \Rightarrow \cos^2 x - \cos x < 0 \Rightarrow 0 < \cos x < 1 \Rightarrow$
$\Rightarrow 0 < x < \frac{\pi}{2}$

444. Fazendo $2x = y \Rightarrow \operatorname{tg} y \geqslant -\sqrt{3} \Rightarrow \left(2k\pi \leqslant y < \frac{\pi}{2} + 2k\pi \text{ ou}\right.$

$\left.\frac{2\pi}{3} + 2k\pi \leqslant y < \frac{3\pi}{2} + 2k\pi \text{ ou } \frac{5\pi}{3} + 2k\pi \leqslant y \leqslant 2\pi + 2k\pi\right) \Rightarrow$

$\Rightarrow \left(k\pi \leqslant x < \frac{\pi}{4} + k\pi \text{ ou } \frac{2\pi}{6} + k\pi \leqslant x < \frac{3\pi}{4} + k\pi \text{ ou}\right.$

$\left.\frac{5\pi}{6} + k\pi \leqslant x \leqslant \pi + k\pi\right) \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \frac{\pi}{4} \text{ ou } \frac{\pi}{3} \leqslant x < \frac{3\pi}{4} \text{ ou } \frac{5\pi}{6} \leqslant x < \frac{5\pi}{4} \text{ ou}\right.$

$\left.\frac{4\pi}{3} \leqslant x < \frac{7\pi}{4} \text{ ou } \frac{11\pi}{6} \leqslant x \leqslant 2\pi\right\}$

445. $\operatorname{tg}^2 2x - \operatorname{tg} 2x \leqslant 0 \Rightarrow 0 \leqslant \operatorname{tg} 2x \leqslant 1 \Rightarrow$

$\Rightarrow k\pi \leqslant 2x \leqslant \frac{\pi}{4} + k\pi \Rightarrow \frac{k\pi}{2} \leqslant x \leqslant \frac{\pi}{8} + \frac{k\pi}{2} \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \frac{\pi}{8} \text{ ou } \frac{\pi}{2} \leqslant x \leqslant \frac{5\pi}{8} \text{ ou } \pi \leqslant x \leqslant \frac{9\pi}{8} \text{ ou}\right.$

$\left.\frac{3\pi}{2} \leqslant x \leqslant \frac{13\pi}{8}\right\}$

446. Fazendo $\operatorname{tg} 2x = t \Rightarrow t^2 - 3 < 0 \Rightarrow -\sqrt{3} < t < \sqrt{3} \Rightarrow -\sqrt{3} < \operatorname{tg} 2x < \sqrt{3}$.

Fazendo $2x = y \Rightarrow -\sqrt{3} < \operatorname{tg} y < \sqrt{3} \Rightarrow \frac{2\pi}{3} + 2k\pi < y < \frac{4\pi}{3} + 2k\pi$ ou

$2k\pi \leqslant y < \frac{\pi}{3} + 2k\pi$ ou $\frac{5\pi}{3} + 2k\pi < y < 2\pi + 2k\pi \Rightarrow$

$\Rightarrow \frac{\pi}{3} + k\pi < x < \frac{2\pi}{3} + k\pi$ ou

$k\pi \leqslant x < \frac{\pi}{6} + k\pi$ ou $\frac{5\pi}{6} + k\pi < x < \pi + k\pi \Rightarrow$

$\Rightarrow S = \left\{x \in \mathbb{R} \mid 0 \leqslant x < \dfrac{\pi}{6} \text{ ou } \dfrac{\pi}{3} < x < \dfrac{2\pi}{3} \text{ ou } \dfrac{5\pi}{6} < x < \dfrac{7\pi}{6} \text{ ou}\right.$
$\left.\dfrac{4\pi}{3} < x < \dfrac{5\pi}{3} \text{ ou } \dfrac{11\pi}{6} < x \leqslant 2\pi\right\}$

447. $\text{sen } x - \cos x > 0 \Rightarrow \text{sen } x - \text{sen}\left(\dfrac{\pi}{2} - x\right) > 0 \Rightarrow \text{sen}\left(x - \dfrac{\pi}{4}\right) > 0 \Rightarrow$

$\Rightarrow 0 < x - \dfrac{\pi}{4} < \pi \Rightarrow \dfrac{\pi}{4} < x < \dfrac{5\pi}{4}$

448. $\cos x + \dfrac{\text{sen } \dfrac{\pi}{3}}{\cos \dfrac{\pi}{3}} \cdot \text{sen } x > \sqrt{2} \Rightarrow$

$\Rightarrow \cos x \cdot \cos \dfrac{\pi}{3} + \text{sen } \dfrac{\pi}{3} \cdot \text{sen } x > \sqrt{2} \cos \dfrac{\pi}{3} \Rightarrow$

$\Rightarrow \cos\left(x - \dfrac{\pi}{3}\right) > \dfrac{\sqrt{2}}{2}$ fazendo $x - \dfrac{\pi}{3} = t \Rightarrow \cot t > \dfrac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow \left(0 \leqslant t < \dfrac{\pi}{4} \text{ ou } -\dfrac{\pi}{4} < t \leqslant 0\right)$. E daí: $0 \leqslant x - \dfrac{\pi}{3} < \dfrac{\pi}{4} \Rightarrow$

$\Rightarrow \dfrac{\pi}{3} < x < \dfrac{7\pi}{12}$ ou $-\dfrac{\pi}{4} < x - \dfrac{\pi}{3} \leqslant 0 \Rightarrow \dfrac{\pi}{12} < x \leqslant \dfrac{\pi}{3}$; portanto,

$S = \left\{x \in \mathbb{R} \mid \dfrac{\pi}{12} < x < \dfrac{7\pi}{12}\right\}$

449. $\pi \leqslant x \leqslant 2\pi \Rightarrow \text{sen } x \leqslant 0 \Rightarrow |\cos x| \geqslant \text{sen } x.$

Se $\text{sen } x \geqslant 0$, a inequação equivale a $\cos^2 x \geqslant \text{sen}^2 x$ e daí

$2 \cdot \text{sen}^2 x - 1 \leqslant 0$, portanto $-\dfrac{\sqrt{2}}{2} \leqslant \text{sen } x \leqslant \dfrac{\sqrt{2}}{2}$.

Tendo em vista a hipótese, temos $0 \leqslant \text{sen } x \leqslant \dfrac{\sqrt{2}}{2}$, de onde vem

$0 \leqslant x \leqslant \dfrac{\pi}{4}$ ou $\dfrac{3\pi}{4} \leqslant x \leqslant \pi$.

$S = \left\{x \in \mathbb{R} \mid 0 \leqslant x \leqslant \dfrac{\pi}{4} \text{ ou } \dfrac{3\pi}{4} \leqslant x \leqslant 2\pi\right\}$

450. $\dfrac{2 \text{ tg } x\left(1 + \text{tg}^2 x - \dfrac{1}{3}\right)}{1 + \text{tg}^2 x} \leqslant 0 \Rightarrow \text{tg } x \leqslant 0 \Rightarrow$

$\Rightarrow \dfrac{\pi}{2} < x \leqslant \pi \text{ ou } \dfrac{3\pi}{2} < x \leqslant 2\pi$

451. $\text{sen}^2\, x - \dfrac{1}{4} \geqslant 0 \Rightarrow \text{sen}\, x \geqslant \dfrac{1}{2}$ ou $\text{sen}\, x \leqslant -\dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}$ ou

$\dfrac{7\pi}{6} \leqslant x \leqslant \dfrac{11\pi}{6}$

452. $3^{2\,\text{sen}\, x\, -\, 1} \geqslant 3^0 \Rightarrow 2\, \text{sen}\, x - 1 \geqslant 0 \Rightarrow \text{sen}\, x \geqslant \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} \leqslant x \leqslant \dfrac{5\pi}{6}$

453. a) $2\, \text{sen}\, x - 1 > 0 \Rightarrow \text{sen}\, x > \dfrac{1}{2} \Rightarrow \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

b) $\log_2 (2 \cdot \text{sen}\, x - 1) = \dfrac{1}{2} \cdot \log_2 (3\, \text{sen}^2\, x - 4 \cdot \text{sen}\, x + 2)$

$\log_2 (2 \cdot \text{sen}\, x - 1)^2 = \log_2 (3 \cdot \text{sen}^2\, x - 4 \cdot \text{sen}\, x + 2)$

Então:

$(2 \cdot \text{sen}\, x - 1)^2 = 3 \cdot \text{sen}^2\, x - 4 \cdot \text{sen}\, x + 2 \Rightarrow$

$\Rightarrow \text{sen}^2\, x = 1 \Rightarrow \text{sen}\, x = \pm 1$

Levando em conta a parte a), resulta $\text{sen}\, x = 1 \Rightarrow x = \dfrac{\pi}{2}$.

454. $\cos^2 x - \dfrac{3}{4} > 0 \Rightarrow -\dfrac{\sqrt{3}}{2} < \cos x < \dfrac{\sqrt{3}}{2} \Rightarrow$

$\Rightarrow \dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$ ou $\dfrac{7\pi}{6} < x < \dfrac{11\pi}{6}$

455. Fazendo $\cos x = y \Rightarrow \dfrac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \dfrac{1}{2}$ ou $y > 1$

e daí $-1 < \cos x < \dfrac{1}{2} \Rightarrow \dfrac{\pi}{3} < x < \pi$.

456. $\dfrac{4\, \text{sen}^2\, x - 1}{\cos x} \geqslant 0 \Rightarrow \dfrac{4 - 4\cos^2 x - 1}{\cos x} \geqslant 0$

Fazendo $\cos x = y \Rightarrow \dfrac{3 - 4y^2}{y} \geqslant 0 \Rightarrow \left(y \leqslant -\dfrac{\sqrt{3}}{2} \text{ ou } 0 < y \leqslant \dfrac{\sqrt{3}}{2}\right) \Rightarrow$

$\Rightarrow \left(\cos x \leqslant -\dfrac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leqslant \dfrac{\sqrt{3}}{2}\right) \Rightarrow$

$\Rightarrow \dfrac{\pi}{6} \leqslant x < \dfrac{\pi}{2}$ ou $\dfrac{5\pi}{6} \leqslant x \leqslant \dfrac{7\pi}{6}$ ou $\dfrac{3\pi}{2} < x \leqslant \dfrac{11\pi}{6}$

457. $x^2 + x + \left(\text{tg}\, \alpha - \dfrac{3}{4}\right) > 0, \forall x \Rightarrow \Delta < 0 \Rightarrow 1 - 4\, \text{tg}\, \alpha + 3 < 0 \Rightarrow$

$\Rightarrow \text{tg}\, \alpha > 1 \Rightarrow \dfrac{\pi}{4} < \alpha < \dfrac{\pi}{2}$

458. $\text{sen}^2\, x - 2\,\text{sen}\, x < 0 \Rightarrow 0 < \text{sen}\, x < 2 \Rightarrow 0 < x < \pi$

459. $\text{sen}^2\, x + \cos^2 x + 2\,\text{sen}\, x \cos x > 1 \Rightarrow \text{sen}\, 2x > 0$

Fazendo $2x = t$, temos:

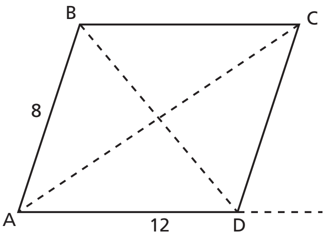
$\text{sen}\, t > 0 \Rightarrow 2k\pi < t < \pi + 2k\pi \Rightarrow k\pi < x < \dfrac{\pi}{2} + k\pi \Rightarrow$

$\Rightarrow 0 < x < \dfrac{\pi}{2}$ ou $\pi < x < \dfrac{3\pi}{2}$

Apêndice B — Trigonometria em triângulos quaisquer

461. $c^2 = 4^2 + (3\sqrt{2})^2 - 2 \cdot 4 \cdot 3\sqrt{2} \cdot \cos 45° \Rightarrow c = \sqrt{10}$

462.



$BD = a$ e $AC = b$

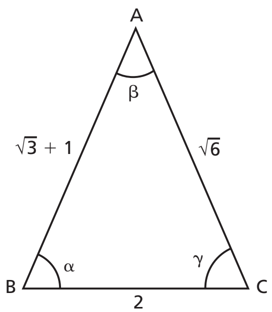
$a^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \cos 60° \Rightarrow$

$\Rightarrow a = 4\sqrt{7}$ m

$b^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \cos 120° \Rightarrow$

$\Rightarrow b = 4\sqrt{19}$ m

463.



$2^2 = (\sqrt{6})^2 + (\sqrt{3} + 1)^2 - 2(\sqrt{6})(\sqrt{3} + 1) \cos \beta \Rightarrow$

$\Rightarrow \cos \beta = \dfrac{\sqrt{2}}{2} \Rightarrow \beta = 45°$

$(\sqrt{6})^2 = (\sqrt{3} + 1)^2 + 2^2 - 2 \cdot 2(\sqrt{3} + 1) \cos \alpha \Rightarrow$

$\Rightarrow \cos \alpha = \dfrac{1}{2} \Rightarrow \alpha = 60°$

$\alpha + \beta + \gamma = 180° \Rightarrow \gamma = 75°$

464. $a, b, c \in \mathbb{Q} \Rightarrow a^2, b^2, c^2 \in \mathbb{Q} \Rightarrow (a^2 + c^2 - b^2) \in \mathbb{Q}$

$a, c \in \mathbb{Q} \Rightarrow 2ac \in \mathbb{Q}$

$\dfrac{a^2 + c^2 - b^2}{2ac} \in \mathbb{Q}$ e $\cos \beta = \dfrac{a^2 + c^2 - b^2}{2ac} \Rightarrow \cos \beta \in \mathbb{Q}$

465. $(x^2 + x + 1)^2 = (x^2 - 1)^2 + (2x + 1)^2 - 2(x^2 - 1)(2x + 1) \cdot \cos \beta \Rightarrow$

$\Rightarrow \cos \beta = \dfrac{2x^3 + x^2 - 2x - 1}{-4x^3 - 2x^2 + 4x + 2} = \dfrac{1 \cdot (2x^3 + x^2 - 2x - 1}{-2 \cdot (2x^3 + x^2 - 2x - 1)} = -\dfrac{1}{2} \Rightarrow$

$\Rightarrow \beta = 120°$

466. $a^2 = c^2 + 1 - 2c\cos 120° \Rightarrow a^2 - c^2 - c = 1 \Rightarrow (2c)^2 - c^2 - c = 1 \Rightarrow$

$\Rightarrow c = \dfrac{1 + \sqrt{13}}{6}$

468. a) $17^2 = 15^2 + 8^2 \Rightarrow$ O triângulo é retângulo.

b) $10^2 > 5^2 + 6^2 \Rightarrow$ O triângulo é obtusângulo.

c) $8^2 < 6^2 + 7^2 \Rightarrow$ O triângulo é acutângulo.

469. Chamando as medidas dos lados de *a*, aq, aq^2, só falta impor duas condições:

(1) o maior lado é menor que a soma dos outros dois (condição para existência do triângulo): $aq^2 < a + aq$;

(2) o quadrado do maior lado é maior que a soma dos quadrados dos outros dois (condição para o triângulo ser obtusângulo): $(aq^2)^2 > a^2 + (aq)^2$.

De (1) resulta $\dfrac{1 - \sqrt{5}}{2} < q < \dfrac{1 + \sqrt{5}}{2}$.

De (2) resulta $q < -\sqrt{\dfrac{1 + \sqrt{5}}{2}}$ ou $q > \sqrt{\dfrac{1 + \sqrt{5}}{2}}$.

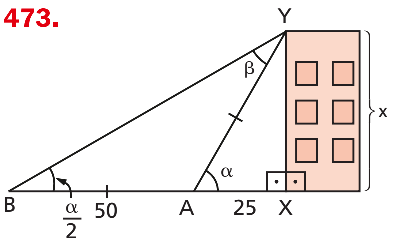
Como $q > 0$, temos $\sqrt{\dfrac{1 + \sqrt{5}}{2}} < q < \dfrac{1 - \sqrt{5}}{2}$.

472.

$$\left.\begin{array}{l} \text{sen}\,\hat{B} = \dfrac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60° \text{ ou } \hat{B} = 120° \\ \text{sen}\,\hat{C} = \dfrac{\sqrt{2}}{2} \Rightarrow \hat{C} = 45° \text{ ou } C = 135° \\ A + B + C = 180° \Rightarrow B + C = 180° - 15° \Rightarrow B + C = 165° \end{array}\right\} \Rightarrow$$

$\Rightarrow \hat{B} = 120°$ e $\hat{C} = 45°$ e $\hat{A} = 15°$

473.



$\alpha = \dfrac{\alpha}{2} + \beta$ (ângulo externo ao $\triangle ABY$) $\Rightarrow$

$\Rightarrow \beta = \dfrac{\alpha}{2}$

$\therefore$ o triângulo ABY é isósceles $\Rightarrow$

$\Rightarrow BA = AY = 50$ m

No $\triangle AXY$ temos $XY^2 = AY^2 - AX^2 =$

$= 50^2 - 25^2$

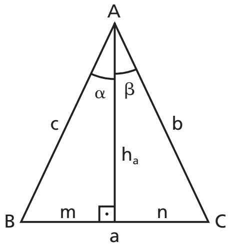
$XY = 25\sqrt{3}$ m

474.

$$\left.\begin{aligned} \text{tg}\,\alpha = \frac{m}{h_a} \Rightarrow h_a = \frac{m}{\text{tg}\,\alpha} \\ \text{tg}\,\beta = \frac{n}{h_a} \Rightarrow h_a = \frac{n}{\text{tg}\,\beta} \end{aligned}\right\} \Rightarrow \frac{m}{\text{tg}\,\alpha} = \frac{n}{\text{tg}\,\beta} \Rightarrow$$

$$\Rightarrow \frac{m+n}{\text{tg}\,\alpha + \text{tg}\,\beta} = \frac{m}{\text{tg}\,\alpha} \Rightarrow \frac{a}{\text{tg}\,\alpha + \text{tg}\,\beta} = \frac{m}{\text{tg}\,\alpha} \Rightarrow$$

$$\Rightarrow a = \frac{m}{\text{tg}\,\alpha} \cdot (\text{tg}\,\alpha + \text{tg}\,\beta) \Rightarrow a = h_a(\text{tg}\,\alpha + \text{tg}\,\beta)$$

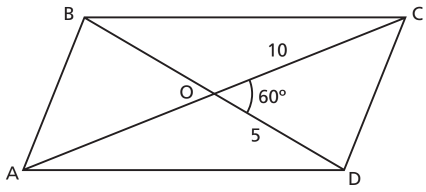


475. $S = \dfrac{4 \cdot 7 \text{ sen } 60°}{2} \Rightarrow S = 7\sqrt{3} \text{ m}^2$

476. Sabe-se que as diagonais de um paralelogramo dividem-se mutuamente ao meio, então:

$\overline{AO} = \overline{OC} = 10$ m e

$\overline{BO} = \overline{OD} = 5$ m



Além disso, as diagonais dividem o paralelogramo em quatro triângulos de áreas iguais, então:

$$S_{ABCD} = 4 \cdot S_{DOC} = 4 \cdot \frac{\overline{DO} \cdot \overline{OC} \cdot \text{sen } D\hat{O}C}{2} = 4 \cdot \frac{5 \cdot 10 \cdot \sqrt{3}}{45} = 50\sqrt{3}$$

477. $S = \dfrac{8 \cdot 10}{2} \text{ sen } \alpha \Rightarrow \text{sen } \alpha = \dfrac{1}{2} \Rightarrow \alpha = 30°,$

$a^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cdot \cos 30° \Rightarrow a = 2\sqrt{41 - 20\sqrt{3}},$

$$\frac{a}{\text{sen } \alpha} = 2R \Rightarrow \frac{2\sqrt{41-20\sqrt{3}}}{\frac{1}{2}} = 2R \Rightarrow R = 2\sqrt{41-20\sqrt{3}} \text{ (m)}$$

478. $7^2 = c^2 + 8^2 - 2 \cdot 8 \cdot c \cdot \cos 60° \Rightarrow c^2 - 8c + 15 = 0 \Rightarrow c = 5$ ou $c = 3$

$c = 5 \text{ m} \Rightarrow S = \dfrac{5 \cdot 8}{2} \text{ sen } 60° \Rightarrow S = 10\sqrt{3} \text{ m}^2$ ou $c = 3 \text{ m} \Rightarrow S = 6\sqrt{3} \text{ m}^2$

479. $A + B + C = 180° \Rightarrow A + B = 180° - C$ ou $B + C = 180° - A,$

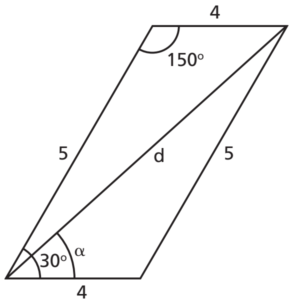
$\cos(180° - C) = -\cos C \Rightarrow \cos(A + B) = \cos(180° - C) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \cos C = -\dfrac{1}{2} \Rightarrow C = 120°$

$\text{sen}(B + C) = \text{sen}(180° - A) = \text{sen } A \Rightarrow \text{sen } A = \dfrac{1}{2} \Rightarrow A = 30°$

$B = 180° - (A + C) = 30°$

480.



$$d^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 150° = $$
$$= 41 + 20\sqrt{3}$$

$$\frac{5}{\text{sen } \alpha} = \frac{\sqrt{41 + 20\sqrt{3}}}{\text{sen } 150°} \Rightarrow$$

$$\Rightarrow \text{sen } \alpha = \frac{5}{2\sqrt{41 + 20\sqrt{3}}}$$

$$\alpha = \text{arc sen } \frac{5}{2\sqrt{41 + 20\sqrt{3}}}$$

482. $\frac{a}{5} = \frac{b}{7} = \frac{c}{9} = k \Rightarrow a = 5k, b = 7k, c = 9k$

$$\cos \hat{B} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{25k^2 + 81k^2 - 49k^2}{2(5k)(9k)} = \frac{57}{90} = \frac{19}{30}$$

$$\hat{B} = \text{arc cos } \frac{19}{30}$$

483. $\text{sen } 15° = \text{sen}(45° - 30°) = \text{sen } 45° \cos 30° - \text{sen } 30° \cos 45° = \frac{\sqrt{6} - \sqrt{2}}{4}$

$$\frac{1}{\text{sen } 15°} = \frac{\sqrt{3} + 1}{\text{sen } \hat{B}} \Rightarrow \text{sen } \hat{B} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45° \text{ ou } \hat{B} = 135°$$

$$\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{B} + \hat{C} = 180° - 15° = 165° \Rightarrow$$

$$\Rightarrow (\hat{C} = 120° \text{ e } \hat{B} = 45°) \text{ ou } (\hat{C} = 30° \text{ e } \hat{B} = 135°)$$

484. $c^2 = (2b)^2 + b^2 - 2 \cdot 2b \cdot b \cdot \cos 60° \Rightarrow c^2 = 3b^2 \Rightarrow c = b\sqrt{3}$

$$\frac{c}{\text{sen } 60°} = \frac{b}{\text{sen } \hat{B}} \Rightarrow \text{sen } \hat{B} = \frac{1}{2} \Rightarrow \hat{B} = 30° \text{ (pois } \hat{B} < 120°)$$

$$\hat{A} = 180° - (\hat{B} + \hat{C}) = 90°$$

485. $\hat{B} = 180° - (\hat{A} + \hat{C}) = 180° - 3\hat{A} \Rightarrow \text{sen } \hat{B} = \text{sen } (180° - 3\hat{A}) = \text{sen } 3\hat{A}$

$$\frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} \Rightarrow \frac{b}{c} = \frac{\text{sen } 3\hat{A}}{\text{sen } 2\hat{A}} = \frac{3 \text{ sen } \hat{A} - 4 \text{ sen}^3 \hat{A}}{2 \text{ sen } \hat{A} \cdot \cos \hat{A}} = \frac{3 - 4 \text{ sen}^2 \hat{A}}{2 \cos \hat{A}} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{\sqrt{3}}\right)^2 = \left(\frac{3 - 4 \text{ sen}^2 \hat{A}}{2 \cos \hat{A}}\right)^2 \Rightarrow 48 \text{ sen}^4 \hat{A} - 56 \text{ sen}^2 \hat{A} + 11 = 0 \Rightarrow$$

$$\Rightarrow \text{sen } \hat{A} = \frac{1}{2} \Rightarrow \hat{A} = 30°, \text{ então } \hat{C} = 60° \text{ e } \hat{B} = 90°$$

486. $\dfrac{a}{\text{sen } \hat{A}} = \dfrac{b}{\text{sen } \hat{B}} \Rightarrow \dfrac{6}{\text{sen } 3\hat{B}} = \dfrac{3}{\text{sen } \hat{B}} \Rightarrow$

$\Rightarrow 2 = \dfrac{\text{sen } 3\hat{B}}{\text{sen } \hat{B}} = \dfrac{3 \text{ sen } \hat{B} - 4 \text{ sen}^3 \hat{B}}{\text{sen } \hat{B}} = 3 - 4 \text{ sen}^2 \text{B} \Rightarrow \text{sen B} = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \hat{B} = 30°, \hat{A} = 90° \text{ e } \hat{C} = 60°, c^2 = b^2 + a^2 - 2ab \cos \hat{C} \Rightarrow$

$\Rightarrow c^2 = 3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 60° \Rightarrow c = 3\sqrt{3} \text{ m}$

487. $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{B} + \hat{C} = 180° - \hat{A} \Rightarrow \text{tg } (\hat{B} + \hat{C}) = -\text{tg } \hat{A} \Rightarrow$

$\Rightarrow \dfrac{\text{tg } \hat{B} + \text{tg } \hat{C}}{1 - \text{tg } \hat{B} \cdot \text{tg } \hat{C}} = -\text{tg } \hat{A} \Rightarrow \dfrac{2 \cdot \text{tg } \hat{A}}{1 - \text{tg } \hat{B} \cdot \text{tg } \hat{C}} = -\text{tg } \hat{A} \Rightarrow$

$\Rightarrow 2 = -1 + \text{tg } \hat{B} \cdot \text{tg } \hat{C} \Rightarrow \text{tg } \hat{B} \cdot \text{tg } \hat{C} = 3$

488. $S = \dfrac{3 \cdot 4 \cdot \text{sen } \alpha}{2} = 6 \cdot \text{sen } \alpha$

$S - 3 = \dfrac{3 \cdot 4 \cdot \text{sen } (\alpha - 60°)}{2} = 6 \cdot \text{sen } (\alpha - 60°)$

Então:

$6 \cdot \text{sen } \alpha - 3 = 6 \cdot \text{sen } (\alpha - 60°) \Rightarrow \text{sen } \alpha - \text{sen } (\alpha - 60°) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow 2 \cdot \text{sen } 30° \cdot \cos (\alpha - 30°) = \dfrac{1}{2} \Rightarrow \cos (\alpha - 30°) = \dfrac{1}{2} \Rightarrow$

$\Rightarrow \alpha - 30° = 60° \Rightarrow \alpha = 90° \Rightarrow S = 6 \cdot \text{sen } \alpha = 6 \text{ m}^2$

Apêndice C — Resolução de triângulos

490. $a^2 = b^2 + c^2 \Rightarrow a^2 - c^2 = 3^2 \Rightarrow (a + c)(a - c) = 9 \Rightarrow$

$\Rightarrow (a + c)\sqrt{3} = 9 \Rightarrow a + c = 3\sqrt{3}$ (1)

É dado que $a - c = \sqrt{3}$ (2). De (1) e (2) resulta $a = 2\sqrt{3}$ e $c = \sqrt{3}$,

então $\text{sen } \hat{B} = \dfrac{b}{a} = \dfrac{\sqrt{3}}{2}$ e $\hat{B} = 60°$.

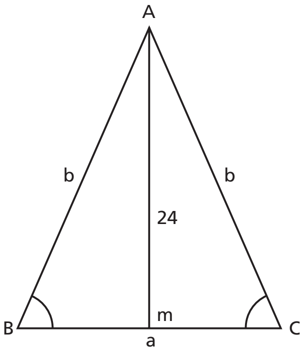
491. Do sistema $a + b = 18$, $a + c = 25$ e $b^2 + c^2 = a^2$, resulta

$(18 - a)^2 + (25 - a)^2 = a^2$ e daí $a = 13$, portanto

$b = 18 - a = 5$ e $c = 25 - a = 12$.

Finalmente $\text{sen } \hat{B} = \dfrac{b}{a} = \dfrac{5}{13}$ e $\text{sen } \hat{C} = \dfrac{c}{a} = \dfrac{12}{13}$.

492.



$2b + a = 64$ (1)

$b^2 = \left(\frac{a}{2}\right)^2 + 24^2$ (2)

De (1) $a = 64 - 2b$, que substituído em (2) dá $b^2 = (32 - b)^2 + 576$ e daí $b = 25$.

(1) $a = 64 - 2b = 14$

$$\cos \hat{B} = \cos \hat{C} = \frac{\frac{a}{2}}{b} = \frac{7}{25}$$

$$\text{sen}\,\frac{\hat{A}}{2} = \frac{\frac{a}{2}}{b} = \frac{7}{25}$$

493. $b = 1,\ c = \text{tg}\,\varphi \Rightarrow a^2 + b^2 = c^2 = 1 + \text{tg}^2\,\varphi = \sec^2\varphi \Rightarrow a = \sec\varphi$

$$\text{tg}\,\hat{B} = \frac{b}{c} = \frac{1}{\text{tg}\,\varphi} = \text{cotg}\,\varphi$$

$$\text{tg}\,\hat{C} = \frac{c}{b} = \text{tg}\,\varphi$$

494. lados: $a,\ b = a + 1,\ c = a + 2$

$$\frac{a}{\text{sen}\,\hat{A}} = \frac{c}{\text{sen}\,\hat{C}} \Rightarrow \frac{a}{\text{sen}\,\hat{A}} = \frac{a+2}{\text{sen}\,2\hat{A}} \Rightarrow \cos\hat{A} = \frac{a+2}{2a}$$

$$a^2 = (a+1)^2 + (a+2)^2 - 2(a+1)(a+2)\cdot\frac{a+2}{2a} \Rightarrow a^2 - 3a - 4 = 0 \Rightarrow$$

$$\Rightarrow a = 4 \text{ e daí: } b = 5,\ c = 6,\ \cos\ \hat{A} = \frac{3}{4},\ \hat{C} = 2\cdot\text{arc sen}\,\frac{\sqrt{7}}{4}$$

495. $$\frac{(a+b+c)\cdot r}{2} = \frac{a\cdot h}{2} \Rightarrow h = \frac{2r^2 + 2ra}{a} \Rightarrow h = \frac{12}{5}$$

$(a + b + c)\cdot r = a\cdot h \Rightarrow b + c = 7$ (A), $b\cdot c = a\cdot h \Rightarrow b\cdot c = 12$ (B)

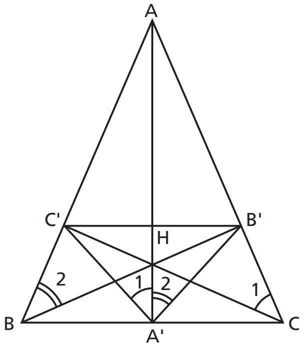
De (A) e (B) vem $b^2 - 7b + 12 = 0 \Rightarrow b = 4$ e $c = 3$ ou $b = 3$ e $c = 4$.

$$\cos\hat{A} = \frac{9 + 16 - 25}{24} = 0 \Rightarrow \hat{A} = 90°,\ \text{sen}\,\hat{B} = \frac{3}{5} \Rightarrow \hat{B} = \text{arc sen}\,\frac{3}{5} \text{ e}$$

$$\cos\hat{C} = \frac{3}{5} \Rightarrow \hat{C} = \text{arc cos}\,\frac{3}{5} \text{ ou sen}\,\hat{B} = \frac{4}{5} \Rightarrow \hat{B} = \text{arc sen}\,\frac{4}{5} \text{ e}$$

$$\cos\hat{C} = \frac{3}{4} \Rightarrow \hat{C} = \text{arc sen}\,\frac{3}{4}$$

496.



Seja H o ponto em que as alturas AA', BB' e CC' se interceptam. Os quadriláteros HA'CB' e HA'BC' são inscritíveis porque têm dois ângulos opostos retos e, portanto, suplementares, então:

$\hat{A}'_1 \equiv \hat{C}_1 \equiv 90° - \hat{A}$ e $\hat{A}'_2 \equiv \hat{B}_2 \equiv 90° - \hat{A}$.

Chamando de $\hat{A}'$, $\hat{B}'$ e $\hat{C}'$ os ângulos do triângulo A'B'C', obtemos:

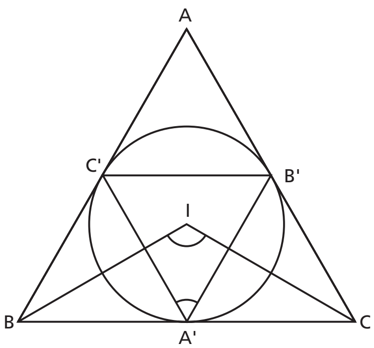
$\hat{A}' \equiv \hat{A}'_1 + \hat{A}'_2 \equiv 180° - 2\hat{A}$, $\hat{B}' \equiv$ $\equiv 180° - 2\hat{B}$ e $\hat{C}' \equiv 180° - 2\hat{C}$.

Aplicando a lei dos senos ao triângulo A'B'C, temos:

$$\frac{A'B'}{\text{sen } C} = \frac{B'C}{\text{sen } B'\hat{A}'C} = \frac{a \cos C}{\text{sen } A} \Rightarrow A'B' = \frac{a \text{ sen } C \cos C}{\text{sen } A}.$$

Analogamente: $B'C' = \dfrac{c \text{ sen } \hat{A} \cos \hat{A}}{\text{sen } B}$ e $A'C' = \dfrac{b \text{ sen } \hat{B} \cos \hat{B}}{\text{sen } C}$.

497.



Seja I o centro da circunferência inscrita em ABC. Ligando I com B e com C, temos:

$$\hat{A}' \equiv 180° - B\hat{I}C \equiv \frac{\hat{B} + \hat{C}}{2}.$$

Analogamente:

$$\hat{B}' \equiv 180° - A\hat{I}C \equiv \frac{\hat{A} + \hat{C}}{2};$$

$$\hat{C}' \equiv 180° - A\hat{I}B \equiv \frac{\hat{A} + \hat{B}}{2}.$$

O triângulo CA'B' é isósceles (CA' ≡ CB'), então

$$c' = A'B' = 2 \cdot CA' \cdot \text{sen } \frac{\hat{C}}{2} = 2(p - c) \text{ sen } \frac{\hat{C}}{2} \text{ em que } p = \frac{a + b + c}{2}.$$

Analogamente, temos:

$$b' = 2(p - b) \text{ sen } \frac{\hat{B}}{2} \text{ e } a' = 2(p - a) \text{ sen } \frac{\hat{A}}{2}.$$

498. $\cos \hat{A} = \sqrt{1 - \text{sen}^2 A} = \dfrac{41}{50}$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \hat{A} \Rightarrow 9 = b^2 + (10 - b)^2 - 2b(10 - b) \cdot \frac{41}{50} \Rightarrow$$
$$\Rightarrow b^2 - 10b + 25 = 0 \Rightarrow b = 5 \Rightarrow c = 5$$

Então $\hat{B} = \hat{C}$ e $\hat{A} + \hat{B} + \hat{C} = \pi$, portanto:

$$\hat{B} = \hat{C} = \frac{\pi}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{1}{2} \cdot \text{arc sen } \frac{3\sqrt{91}}{50}.$$

499. $h_a = c \cdot \text{sen}\,\hat{B} \Rightarrow n = c \cdot \text{sen}\,\hat{B} \Rightarrow c = \dfrac{n}{\text{sen}\,\hat{B}}$

$h_a = b \cdot \text{sen}\,\hat{C} \Rightarrow n = b \cdot \text{sen}\,\hat{C} \Rightarrow b = \dfrac{n}{\text{sen}\,\hat{C}}$

$b + c = m \Rightarrow \dfrac{n}{\text{sen}\,\hat{B}} + \dfrac{n}{\text{sen}\,\hat{C}} = m \ \ (1)$

De (1) vem:

$n\,(\text{sen}\,\hat{B} + \text{sen}\,\hat{C}) = m \cdot \text{sen}\,\hat{B} \cdot \text{sen}\,\hat{C}$

$2n \cdot \text{sen}\,\dfrac{\hat{B}+\hat{C}}{2} \cdot \cos\dfrac{\hat{B}-\hat{C}}{2} = \dfrac{m}{2}\left[\cos\left(\hat{B}-\hat{C}\right) - \cos\left(\hat{B}+\hat{C}\right)\right] \ (2)$

Notemos que:

$\cos\left(\hat{B}+\hat{C}\right) = -\cos\hat{A}$ (dado)

$\text{sen}\,\dfrac{\hat{B}+\hat{C}}{2} = \cos\dfrac{\hat{A}}{2}$ (calculável a partir de $\cos\hat{A}$)

$\cos\left(\hat{B}-\hat{C}\right) = 2 \cdot \cos^2\dfrac{\hat{B}-\hat{C}}{2} - 1$

Então a equação (2) fica:

$m \cdot \cos^2\dfrac{\hat{B}-\hat{C}}{2} - 2n \cdot \cos\dfrac{\hat{A}}{2} \cdot \cos\dfrac{\hat{B}-\hat{C}}{2} - m \cdot \text{sen}^2\dfrac{\hat{A}}{2} = 0$

A partir dessa equação obtém-se o ângulo $\dfrac{\hat{B}-\hat{C}}{2}$.

Como $\dfrac{\hat{B}+\hat{C}}{2} = \dfrac{\pi}{2} - \dfrac{\hat{A}}{2}$, os ângulos $\hat{B}$ e $\hat{C}$ estão determinados e daí

$b = \dfrac{n}{\text{sen}\,\hat{C}},\ c = \dfrac{n}{\text{sen}\,\hat{B}},\ a = \dfrac{m \cdot \text{sen}\left(\hat{B}+\hat{C}\right)}{\text{sen}\,\hat{B} + \text{sen}\,\hat{C}}.$

500. $\dfrac{a}{\text{sen}\,\hat{A}} = \dfrac{b}{\text{sen}\,\hat{B}} = \dfrac{c}{\text{sen}\,\hat{C}} \Rightarrow \dfrac{a+c}{\text{sen}\,\hat{A} + \text{sen}\,\hat{C}} = \dfrac{b}{\text{sen}\,\hat{B}} \Rightarrow$

$\Rightarrow \dfrac{b}{\text{sen}\,(\hat{A}+\hat{C})} = \dfrac{k}{\text{sen}\,\hat{A} + \text{sen}\,\hat{C}} \Rightarrow$

$\Rightarrow \dfrac{b}{2 \cdot \text{sen}\,\dfrac{\hat{A}+\hat{C}}{2} \cdot \cos\dfrac{\hat{A}+\hat{C}}{2}} = \dfrac{k}{2 \cdot \text{sen}\,\dfrac{\hat{A}+\hat{C}}{2} \cdot \cos\dfrac{\hat{A}-\hat{C}}{2}} \Rightarrow$

$\Rightarrow \dfrac{b}{\cos\dfrac{\hat{A}+\hat{C}}{2}} = \dfrac{k}{\cos\dfrac{\hat{A}-\hat{C}}{2}} \Rightarrow$

$\Rightarrow \dfrac{b+k}{\cos\dfrac{\hat{A}+\hat{C}}{2} + \cos\dfrac{\hat{A}-\hat{C}}{2}} = \dfrac{k-b}{\cos\dfrac{\hat{A}-\hat{C}}{2} - \cos\dfrac{\hat{A}+\hat{C}}{2}} \Rightarrow$

$\Rightarrow \dfrac{b+k}{2 \cdot \cos\dfrac{\hat{A}}{2} \cdot \cos\dfrac{\hat{C}}{2}} = \dfrac{k-b}{2 \cdot \text{sen}\,\dfrac{\hat{A}}{2} \cdot \text{sen}\,\dfrac{\hat{C}}{2}} \Rightarrow \text{cotg}\,\dfrac{\hat{C}}{2} = \dfrac{b+k}{k-b} \cdot \text{tg}\,\dfrac{\hat{A}}{2}$

Conhecendo $\hat{C}$, temos:

$B = \pi - \hat{A} - \hat{C}$, $a = \dfrac{b \text{ sen } \hat{A}}{\text{sen } \hat{B}}$ e $c = \dfrac{b \text{ sen } \hat{C}}{\text{sen } \hat{B}}$.

501. $\hat{A} + \hat{B} + \hat{C} = 180° \Rightarrow \hat{A} = 180° - (\hat{B} + \hat{C})$

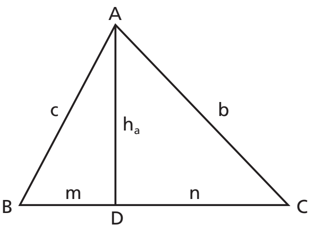
$$\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} = 2R \Rightarrow a = 2R \text{ sen } \hat{A}, b = 2R \text{ sen } \hat{B}, c = 2R \text{ sen } \hat{C}$$

em que R é calculado assim:

$$S = \frac{1}{2} bc \text{ sen } \hat{A} = \frac{1}{2} \cdot 4R^2 \cdot \text{sen } \hat{A} \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}, \text{ então:}$$

$$2R = \sqrt{\frac{2S}{\text{sen } \hat{A} \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}}}$$

502.



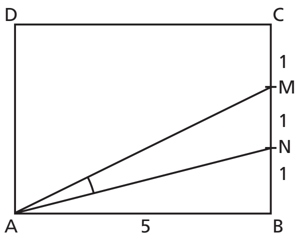
$$a = m + n = \frac{h_a}{\text{tg } \hat{B}} + \frac{h_a}{\text{tg } \hat{C}}$$

$$b = \frac{h_a}{\text{sen } \hat{C}}$$

$$c = \frac{h_a}{\text{sen } \hat{B}}$$

$$\hat{A} = 180° - (\hat{B} + \hat{C})$$

503.

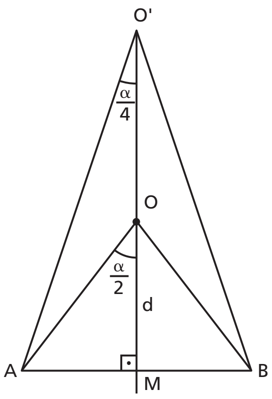


$$\text{tg } M\hat{A}B = \frac{2}{5} \text{ e tg } N\hat{A}B = \frac{1}{5}$$

$$\text{tg } M\hat{A}N = \text{tg } (M\hat{A}B - N\hat{A}B) =$$

$$= \frac{\frac{2}{5} - \frac{1}{5}}{1 + \frac{2}{5} \cdot \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{27}{25}} = \frac{5}{27}$$

504.



$$\frac{AB}{2} = d \cdot \text{tg } \frac{\alpha}{2} \text{ e}$$

$$\frac{AB}{2} = (d + OO') \cdot \text{tg } \frac{\alpha}{4}$$

Então:

$$d \cdot \text{tg } \frac{\alpha}{2} = (d + OO') \cdot \text{tg } \frac{\alpha}{4}$$

$$OO' = \frac{d\left(\text{tg } \frac{\alpha}{2} - \text{tg } \frac{\alpha}{4}\right)}{\text{tg } \frac{\alpha}{4}} = \frac{d}{\cos \frac{\alpha}{2}}$$

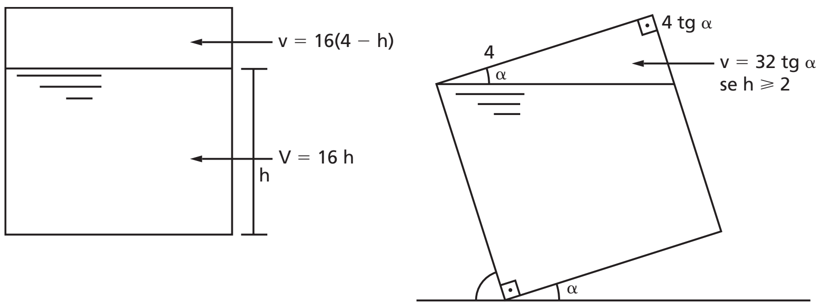
505. $\cos\theta = \dfrac{100 + 49 - 169}{140} = -\dfrac{1}{7} \Rightarrow \sec^2\theta = 49 \Rightarrow 1 + \mathrm{tg}^2\,\theta = 49 \Rightarrow$

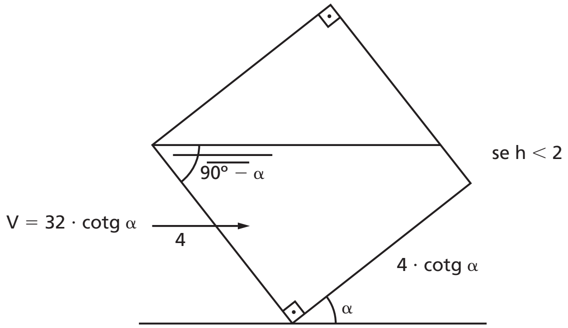
$\Rightarrow \mathrm{tg}\,\theta = -4\sqrt{3} = \alpha$

Então:

$|\sqrt{3}\alpha| = |\sqrt{3}(-4\sqrt{3})| = 12$

506. situação inicial situação final





Se $h \geqslant 2$, então $16(4 - h) = 32\,\mathrm{tg}\,\alpha$ e daí $\mathrm{tg}\,\alpha = \dfrac{4 - h}{2}$.

Se $h < 2$, então $16\,h = 32\,\mathrm{cotg}\,\alpha$ e daí $\mathrm{tg}\,\alpha = \dfrac{2}{h}$.

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